# Nonlinear Beam Theory in Flexible Multibody Dynamics

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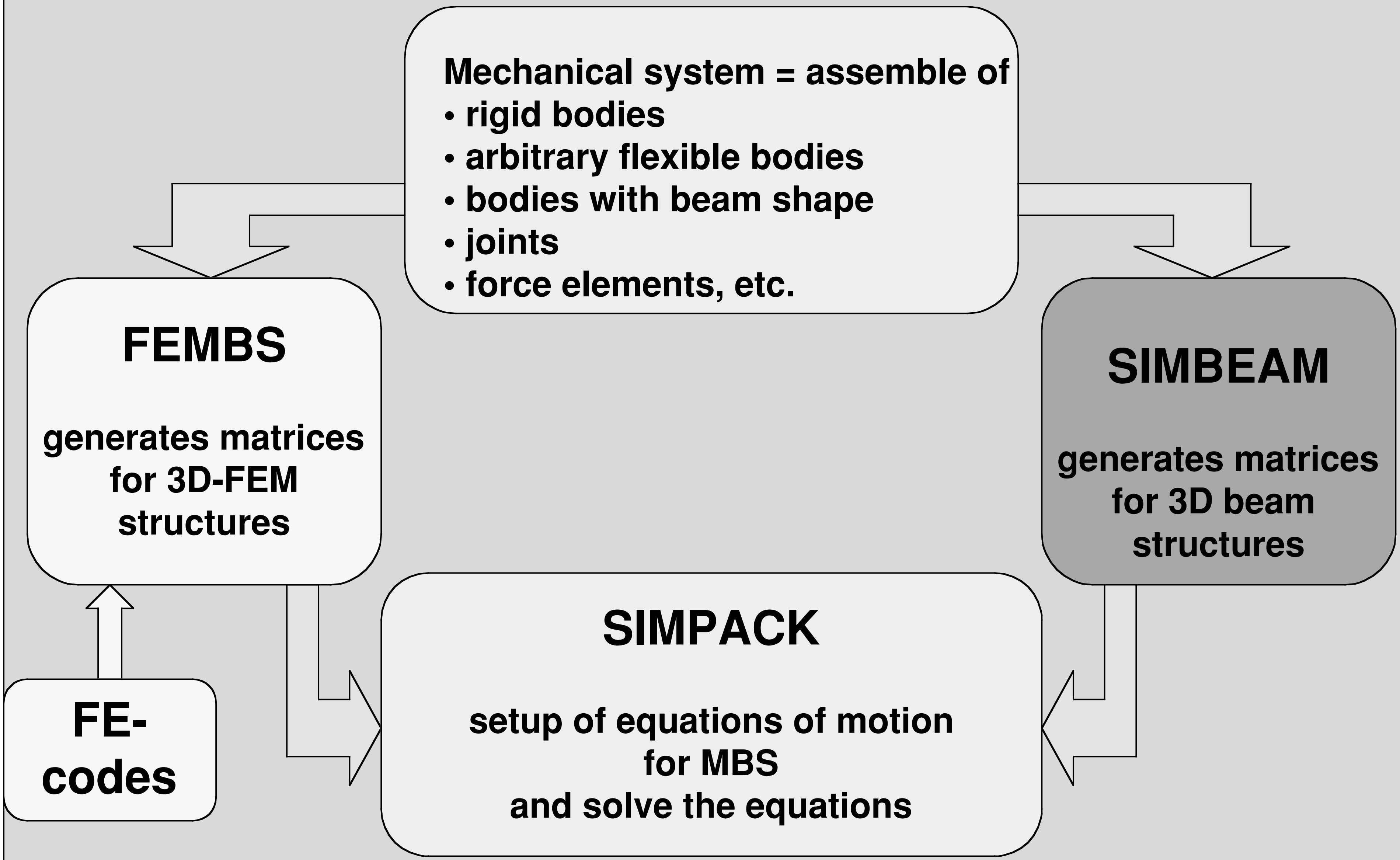
# Multibody Program SIMPACK



# Flexible Body Modeling in MBS

## Body motion =

**a) motion of the floating frame of reference**

**+ deformations using modal or nodal approach**

(general codes like ADAMS, DADS, MEDYNA, SIMPACK,….)

**b) absolute motion of material points using nodal approach**

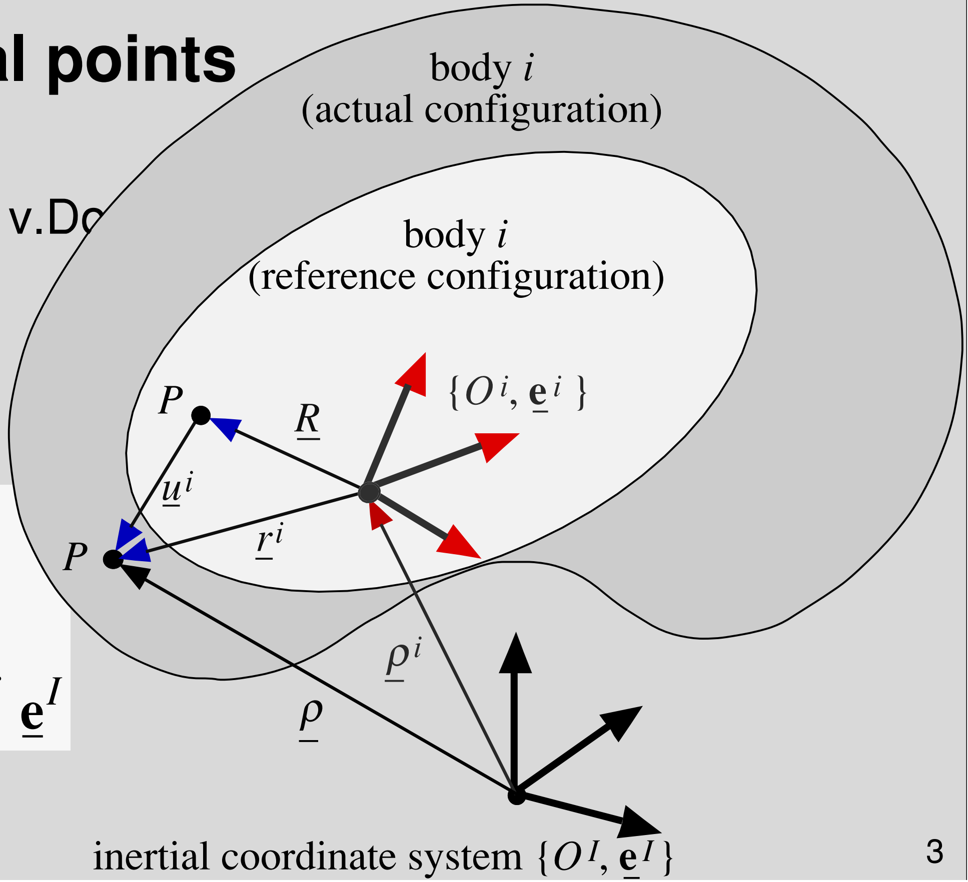
(Ambrosio, Geradin, Simo, Shabana, v.Do

$$\boldsymbol{\rho}(\mathbf{R}) = \boldsymbol{\rho}^i + \mathbf{r}^i(\mathbf{R}) = \boldsymbol{\rho}^i + \mathbf{R} + \mathbf{u}^i(\mathbf{R})$$

$$\mathbf{A}(\mathbf{R}) = \overleftrightarrow{\boldsymbol{\Theta}}^i(\mathbf{R})\,\mathbf{A}^i$$

satisfying

$$\underline{\mathbf{e}} = \mathbf{A}(\mathbf{R})\,\underline{\mathbf{e}}^I, \qquad \underline{\mathbf{e}} = \overleftrightarrow{\boldsymbol{\Theta}}^i(\mathbf{R})\,\underline{\mathbf{e}}^i, \qquad \underline{\mathbf{e}}^i = \mathbf{A}^i\,\underline{\mathbf{e}}^I$$



# ? Design of SIMBEAM

1. What is an efficient formulation
   for the equations of flexible body motion
   in fast simulations of MBS dynamics?

SIMPACK prefers formulations
using floating frame of reference!

2. What kind of approximation for the deformations
   and strains of a beam element should be used?

3. What values of errors occur for typical load cases
   of beam structures?

# Following Content

1. Methods of approximation for deformation and strain

2. Test on a cantilever beam with typical load cases

3. Design of SIMBEAM and Conclusion

# Approximation of Deformations and Strains

(A) Small deformations and small strains are linear in coordinates of deformation $\mathbf{q}^i(t)$.

(B) Small deformations and small strains are linear in $\mathbf{q}^i(t)$ but considering of geometric stiffness matrices.

(C) Small strains are linear in $\mathbf{q}^i(t)$, but displacement field is quadratic in strain variables

(D) Linear displacement field approximation but strains are quadratic in displacement variables

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(C) Small strains are linear in $\mathbf{q}^i(t)$, but displacement field is quadratic in strain variables

(D) Linear displacement field approximation but strains are quadratic in displacement variables

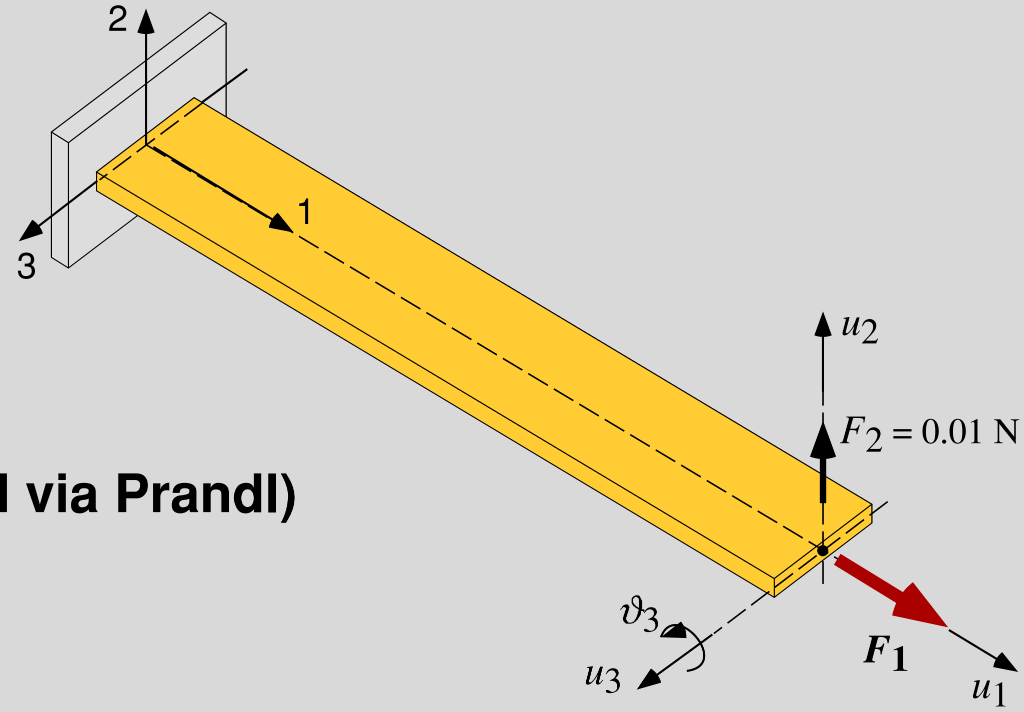
(E) Large deformations and large strains using FE-discretization; e.g. for beam structures (S.v.Dombrowski)

# Approximation of Deformation and Strain

| Model | Displacements | Internal Forces | Inertia Forces |
|---|---|---|---|
| | | | |
| A | $\mathbf{u}^i = \mathbf{\Phi}^i\, \mathbf{q}^i$<br>$\boldsymbol{\vartheta}^i = \mathbf{\Psi}^i\, \mathbf{q}^i$<br>linear | $\delta\boldsymbol{\varepsilon}^i = \mathbf{B}^i\, \delta\mathbf{q}^i$<br>$\boldsymbol{\sigma}^i = \mathbf{H}^i\, \mathbf{B}^i\, \mathbf{q}^i$<br>$\mathbf{k}^i = \mathbf{K}_e^i\, \mathbf{q}^i$<br>linear | $\mathbf{M}^i$ and $\mathbf{h}_\omega^i$ are quadratic in $\mathbf{q}^i$ |
| B | $\mathbf{u}^i = \mathbf{\Phi}^i\, \mathbf{q}^i$<br>$\boldsymbol{\vartheta}^i = \mathbf{\Psi}^i\, \mathbf{q}^i$<br>linear | $\delta\varepsilon_\alpha^i = (\mathbf{B}_{L\alpha}^i + \mathbf{q}^{iT}\mathbf{B}_{Q\alpha}^i)\,\delta\mathbf{q}^i$<br>$\boldsymbol{\sigma}^i = \mathbf{H}^i\, \mathbf{B}^i\, \mathbf{q}^i + \boldsymbol{\sigma}_0^i$<br>$\mathbf{k}^i = (\mathbf{K}_e^i + \mathbf{K}_{geo}^i(\boldsymbol{\sigma}_0^i))\, \mathbf{q}^i$ | $\mathbf{M}^i$ and $\mathbf{h}_\omega^i$ are quadratic in $\mathbf{q}^i$ |
| C | $\boldsymbol{\chi}^i = \mathbf{N}^i\, \mathbf{q}^i$<br>$u_\alpha^i = (\mathbf{\Phi}_{L\alpha}^i + \frac{1}{2}\mathbf{q}^{iT}\mathbf{\Phi}_{Q\alpha}^i)\,\mathbf{q}^i$<br>$\vartheta_\alpha^i = (\mathbf{\Psi}_{L\alpha}^i + \frac{1}{2}\mathbf{q}^{iT}\mathbf{\Psi}_{Q\alpha}^i)\,\mathbf{q}^i$<br>quadr. | $\delta\boldsymbol{\varepsilon}^i = \mathbf{B}^i\, \delta\mathbf{q}^i$<br>$\boldsymbol{\sigma}^i = \mathbf{H}^i\, \mathbf{B}^i\, \mathbf{q}^i$<br>$\mathbf{k}^i = \mathbf{K}_e^i\, \mathbf{q}^i$<br>linear | $\mathbf{M}^i$ and $\mathbf{h}_\omega^i$ are function in $\mathbf{q}^i$ up to the fourth order |
| D | $\mathbf{u}^i = \mathbf{\Phi}^i\, \mathbf{q}^i$<br>$\boldsymbol{\vartheta}^i = \mathbf{\Psi}^i\, \mathbf{q}^i$<br>linear | $\delta\varepsilon_\alpha^i = (\mathbf{B}_{L\alpha}^i + \mathbf{q}^{iT}\mathbf{B}_{Q\alpha}^i)\,\delta\mathbf{q}^i$<br>$\sigma_\alpha^i = H_{\alpha\alpha}^i(\mathbf{B}_{L\alpha}^i + \frac{1}{2}\mathbf{q}^{iT}\mathbf{B}_{Q\alpha}^i)\mathbf{q}^i\,\mathbf{k}^i$<br>$\mathbf{k}^i(\mathbf{q}^i)$ is cubic in $\mathbf{q}^i$<br>quadr. | $\mathbf{M}^i$ and $\mathbf{h}_\omega^i$ are quadratic in $\mathbf{q}^i$ |

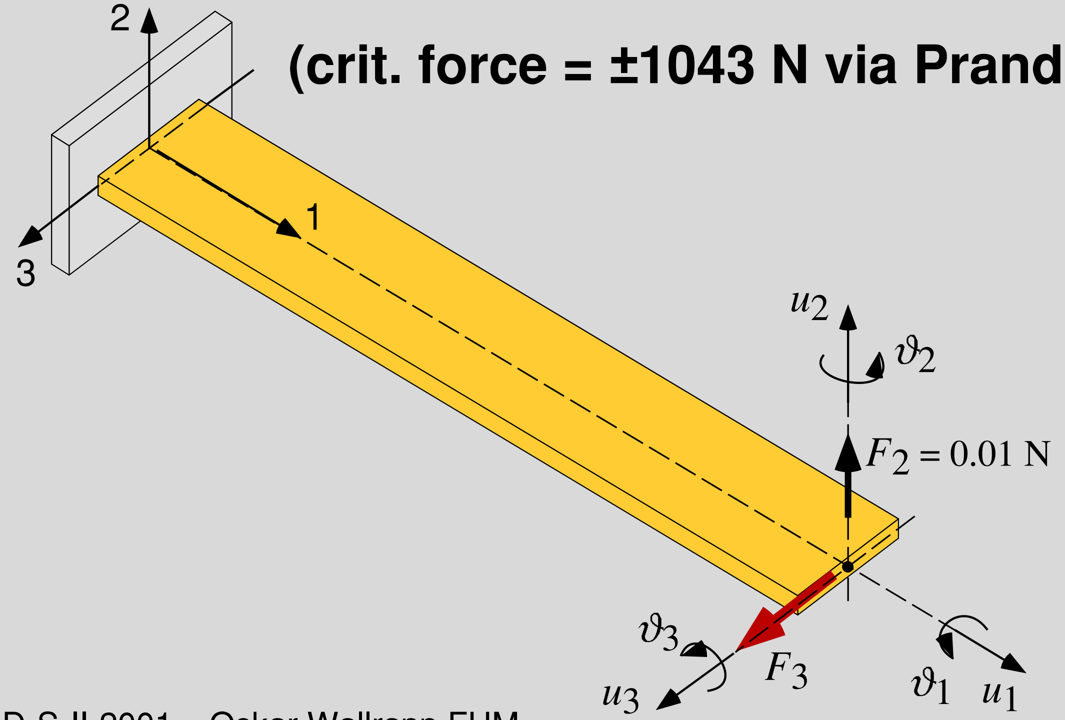
# Examples to Test the Approximations

**Load case 1 – Buckling (crit. force = –525 N via Euler)**



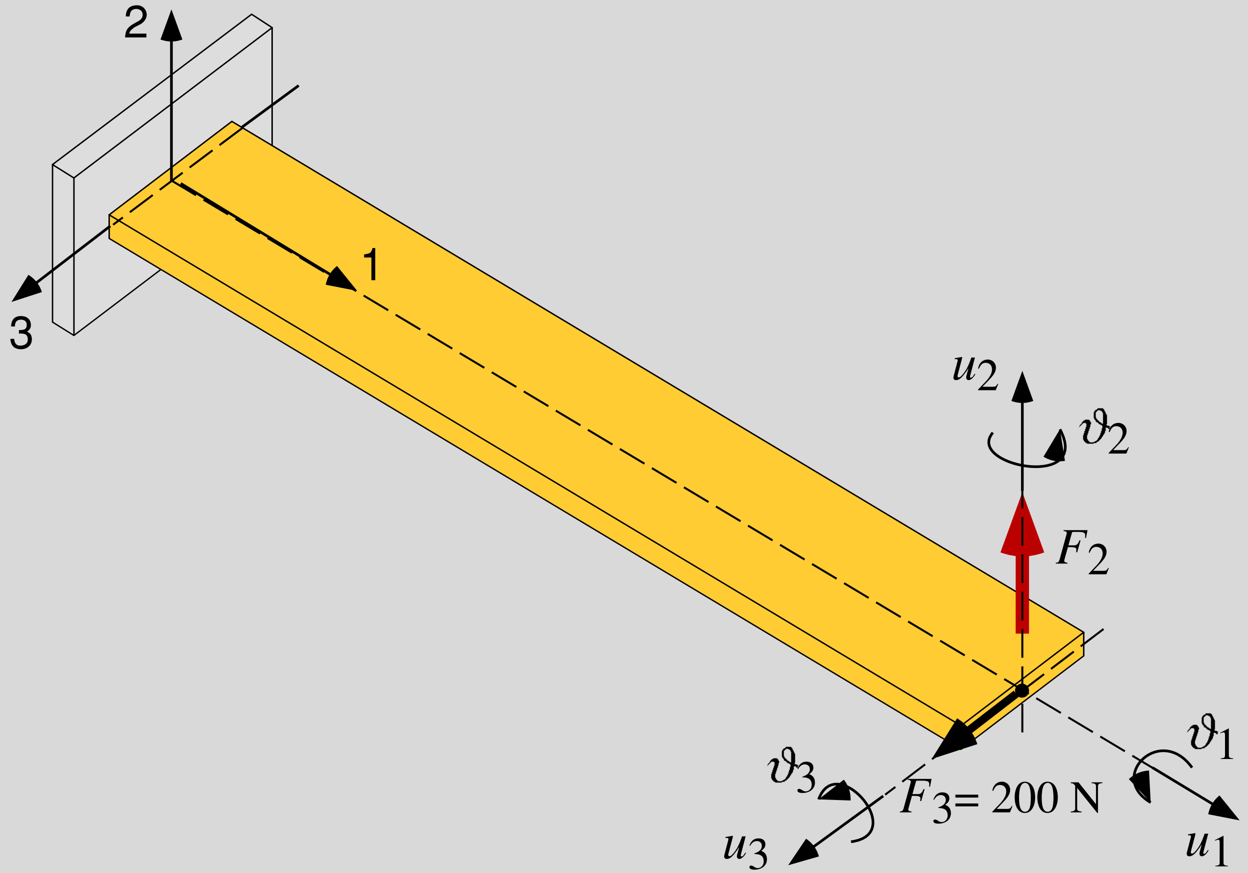

**Load case 2 – Tilting**

**(crit. force = ±1043 N via Prandl)**



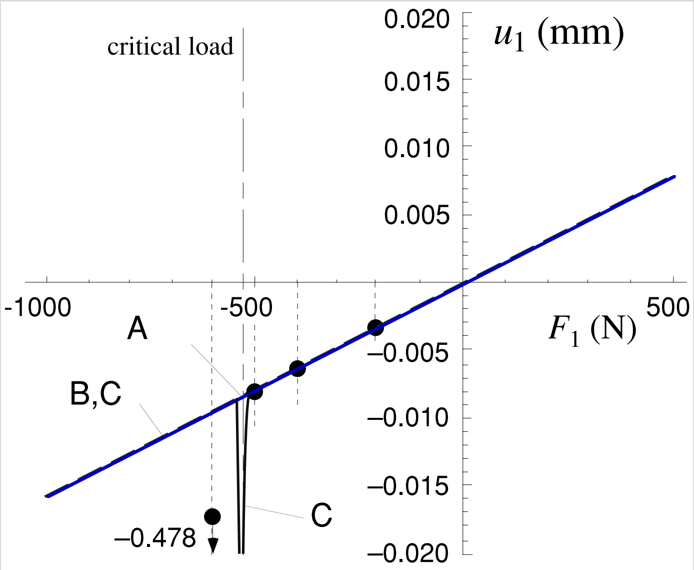
# Examples to Test the Approximations

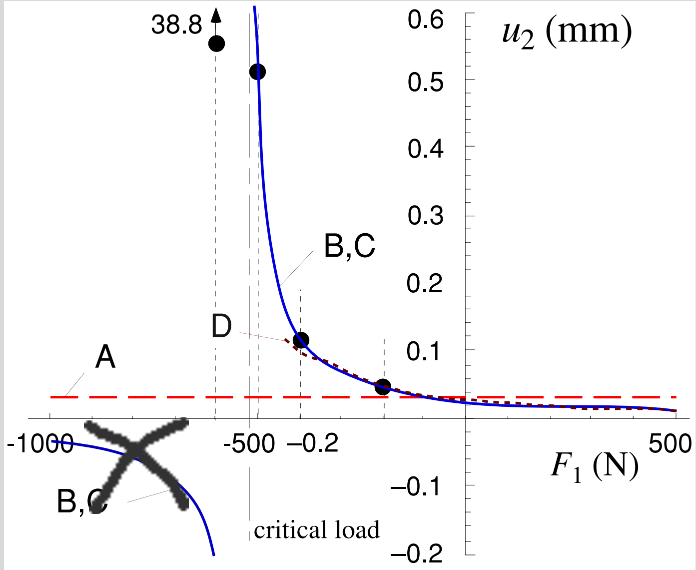
**Load case 3 – Bending in soft direction (quadratic effects for $u_1$, $u_3$, $\vartheta_2$)**



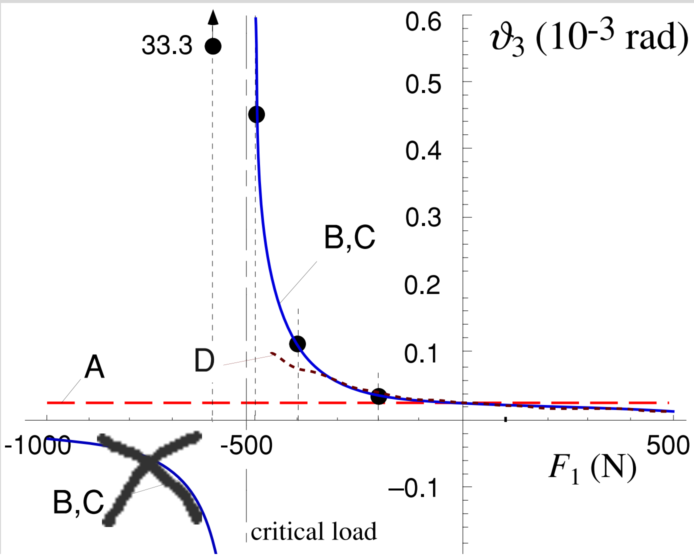
# Solution of the Test Examples

## Load case 1 – Buckling (crit. force = –525 N via Euler)




• = (E)




Results:

(A) $u_2$, $\vartheta_3$ independent of $F_1$, no buckling

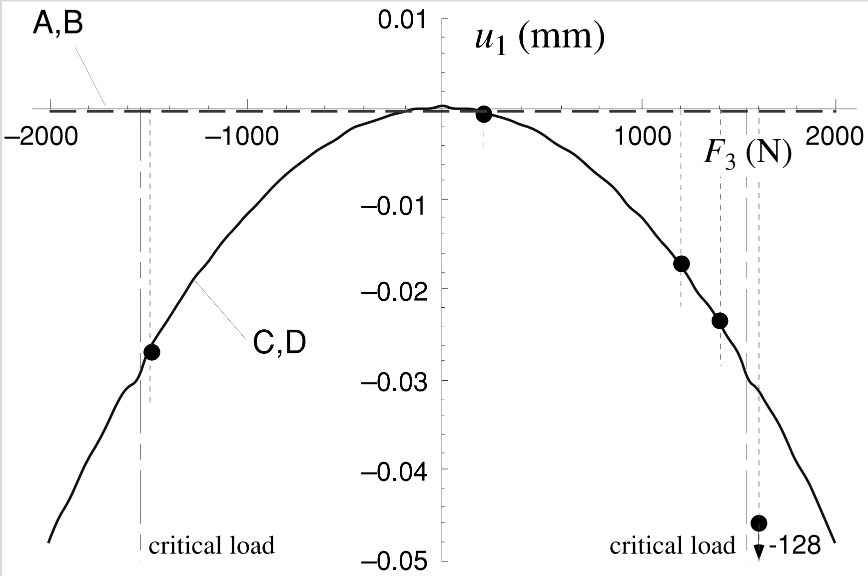
(B),(C) displacements are correct for $F_1 > F_{crit}$

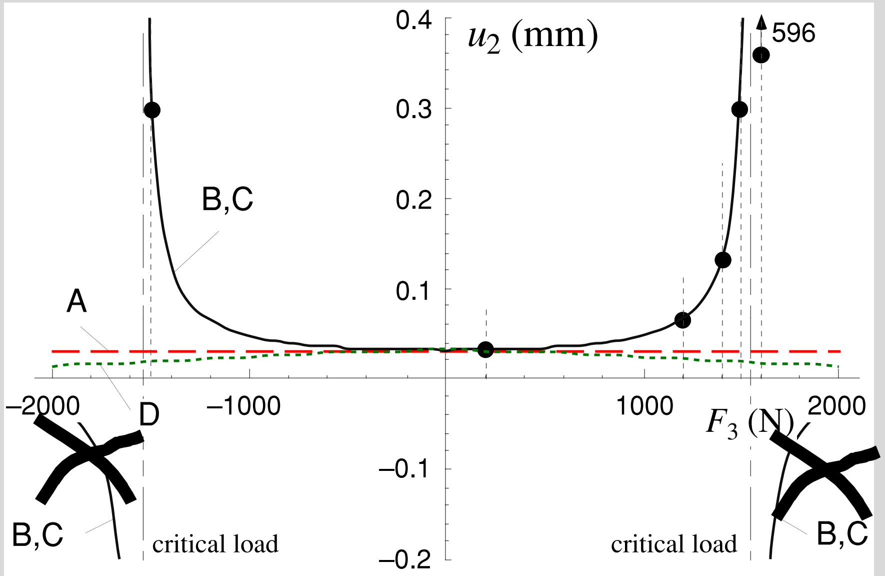
(D) solution failed for $F_1 < 1/2\, F_{crit}$

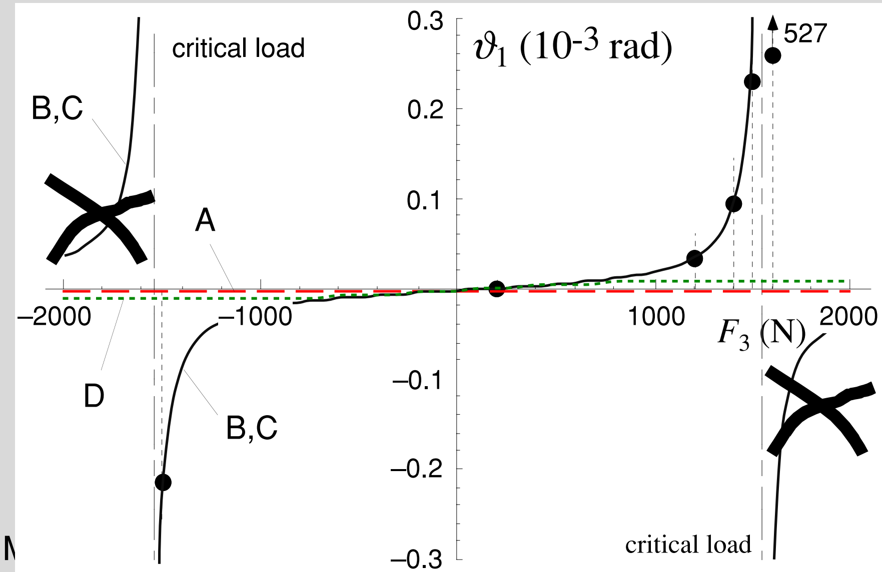
# Solution of the Test Examples

• = (E)

## Load case 2 – Tilting (crit. force = ±1043 N via Prandl, 1560 N for 1 El.)




Results:

(A) $u_1$, $u_2$, $\vartheta_1$, $\vartheta_3$ independent of $F_3$, no tilting

(B) $u_1$ independent of $F_3$, no shortening

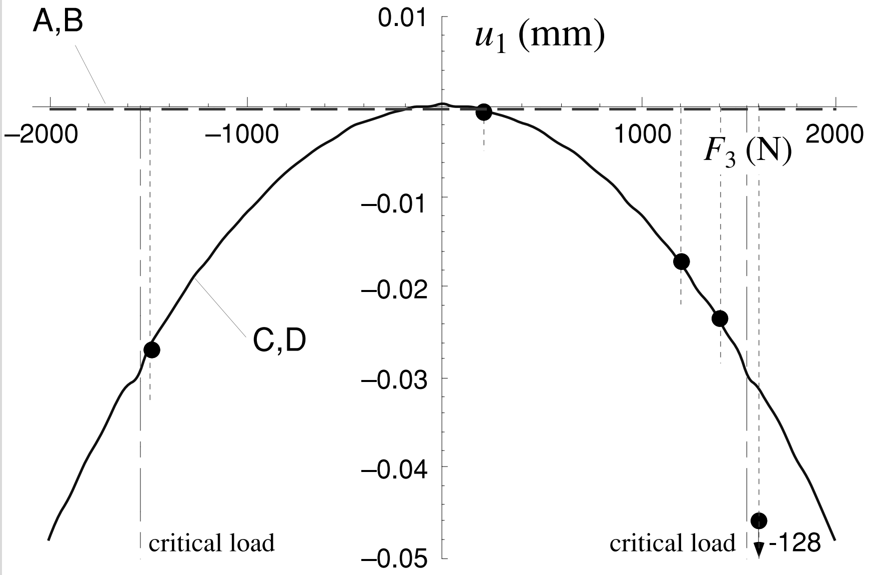
(B),(C) displacements correct (< 1 %) for $|F_3| < F_{crit}$

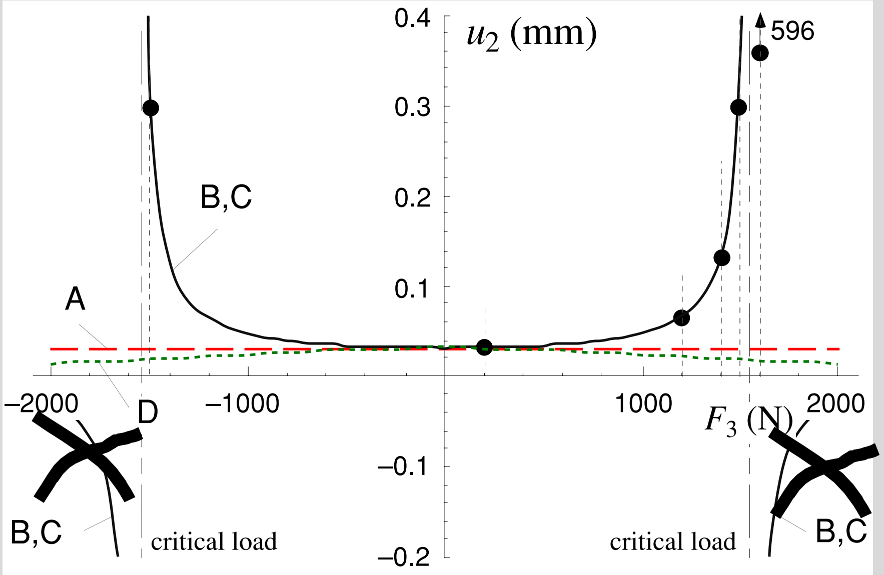
(D) solution failed

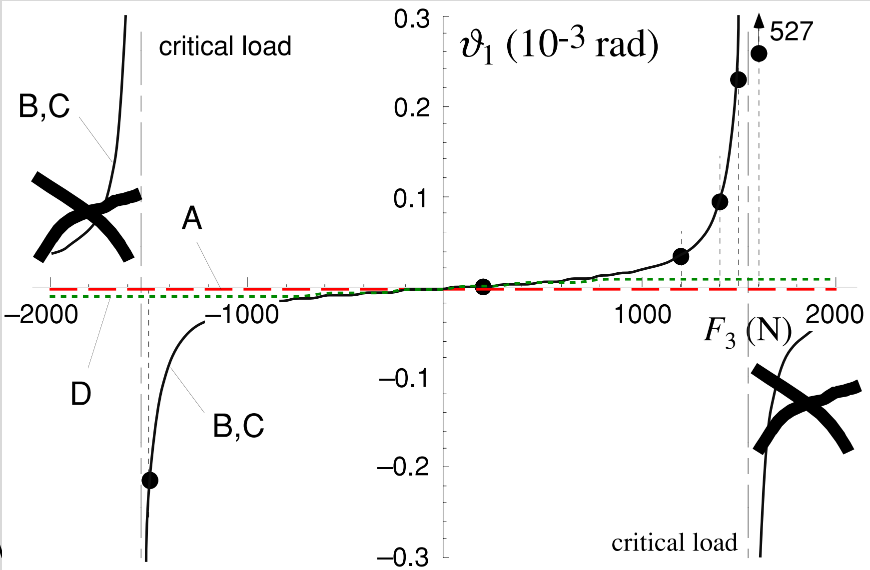
# Solution of the Test Examples

• = (E)

## Load case 2 – Tilting (crit. force = ±1043 N via Prandl, 1560 N for 1 El.)




Results:

(A) $u_1$, $u_2$, $\vartheta_1$, $\vartheta_3$ independent of $F_3$,
no tilting

(B) $u_1$ independent of $F_3$, no shortening

(B),(C) displacements correct (< 1 %)
for $|F_3| < F_{crit}$

(D) solution failed

# Design of SIMBEAM

1. **For 3D-beam structures**
   **model (B) considers buckling and tilting**
   (B) = linear displacement approximation + geometric stiffening matrices

2. **Quadratic effects of shortening and bending coupling ($u_1$, $u_3$, $\vartheta_2$) are considered by displacement corrections due to a quadratic fcts. in $\mathbf{q}^i(t)$ as given in model (C) :**

$$u_\alpha^i(x,t) = \mathbf{\Phi}_{L\alpha}^i(x)\,\mathbf{q}^i + \frac{1}{2}\mathbf{q}^{i^T}\,\mathbf{\Phi}_{Q\alpha}^i(x)\,\mathbf{q}^i$$

3. **Matrices $\mathbf{\Phi}_{Q\alpha}^i$ are derived from quadric strain-displacement relation and pre-computed in SIMBEAM.**

4. **For data transfer, SIMPACK uses SID technology.**

⇨ **SIMBEAM uses items 1, 2, 3, 4**

# Conclusion

**For flexible MBS using floating frame of reference**

1. **a linear approximation of displacement field**

   $$\mathbf{u}(\mathbf{R},t) = \mathbf{\Phi}_L(\mathbf{R})\ \ \mathbf{q}(t)$$

   **by incorporating geometric stiffening effects allows to analyze buckling and tilting problems**

   **but !**
   **never use results for loads higher than the critical load**

2. **shortening and bending coupling will be appear by quadratic correction terms** $+\ 1/2\ \ \mathbf{q}^{\mathrm{T}}\ \mathbf{\Phi}_{Q\alpha}\,\mathbf{q}$ **of displacement field**

3. **the errors of results are in an acceptable range**

# Conclusion

**SIMBEAM is a module of SIMPACK**

4. **satisfying the above items of approximation**
5. **prepare the coefficients of system matrices of a flexible body**
6. **the body is a 3D-FE-net of beam elements**
7. **useful for nodal and modal approach**

**Thank you for your attention !**