Nonlinear Beam Theory in Flexible Multibody Dynamics

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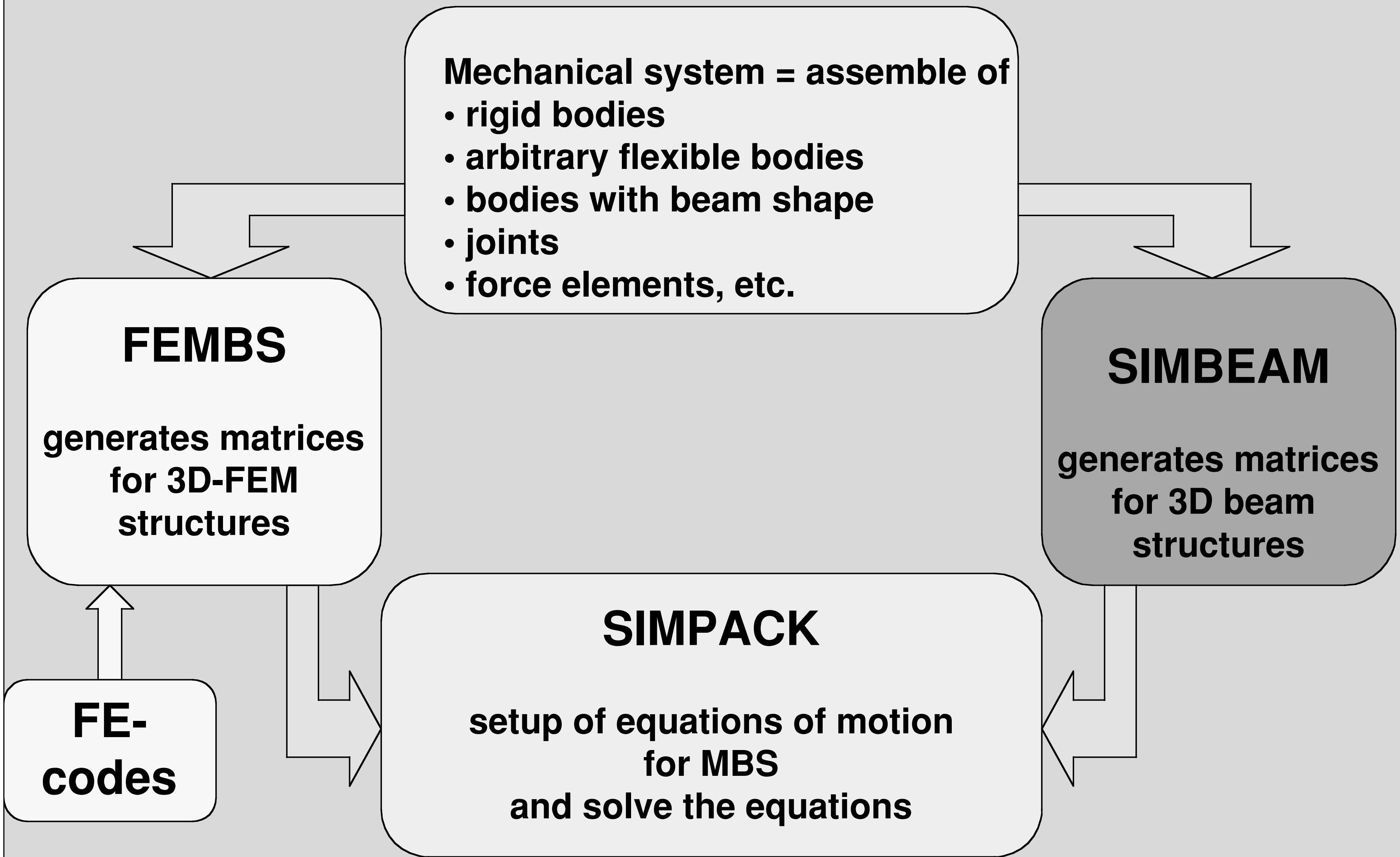
Multibody Program SIMPACK



Flexible Body Modeling in MBS

Body motion =

a) motion of the floating frame of reference

+ deformations using modal or nodal approach

(general codes like ADAMS, DADS, MEDYNA, SIMPACK,….)

b) absolute motion of material points using nodal approach

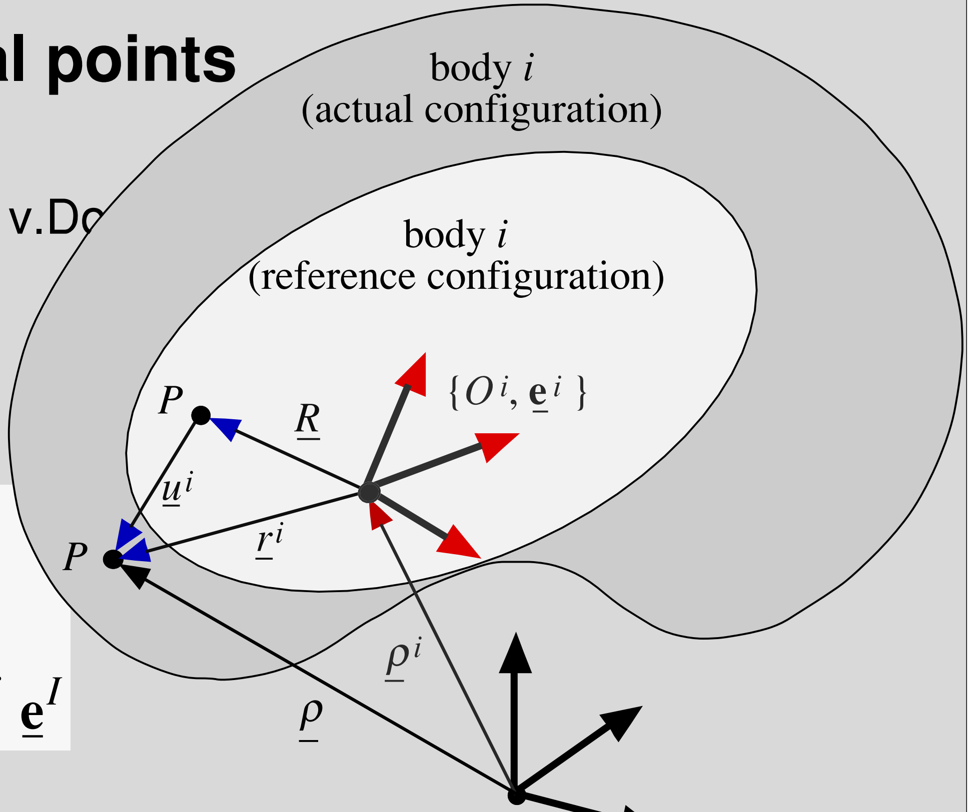
(Ambrosio, Geradin, Simo, Shabana, v.Do

$$\boldsymbol{\rho}(\mathbf{R}) = \boldsymbol{\rho}^i + \mathbf{r}^i(\mathbf{R}) = \boldsymbol{\rho}^i + \mathbf{R} + \mathbf{u}^i(\mathbf{R})$$

$$\mathbf{A}(\mathbf{R}) = \overset{\leftrightarrow}{\boldsymbol{\Theta}}{}^i(\mathbf{R})\,\mathbf{A}^i$$

satisfying

$$\underline{\mathbf{e}} = \mathbf{A}(\mathbf{R})\,\underline{\mathbf{e}}^I, \qquad \underline{\mathbf{e}} = \overset{\leftrightarrow}{\boldsymbol{\Theta}}{}^i(\mathbf{R})\,\underline{\mathbf{e}}^i, \qquad \underline{\mathbf{e}}^i = \mathbf{A}^i\,\underline{\mathbf{e}}^I$$



inertial coordinate system $\{O^I, \underline{\mathbf{e}}^I\}$

? Design of SIMBEAM

1. **What is an <u>efficient</u> formulation for the equations of flexible body motion in fast simulations of MBS dynamics?**

SIMPACK prefers formulations using floating frame of reference!

2. **What kind of <u>approximation</u> for the deformations and strains of a beam element should be used?**

3. **What values of <u>errors</u> occur for typical load cases of beam structures?**

Following Content

1. Methods of approximation for deformation and strain

2. Test on a cantilever beam with typical load cases

3. Design of SIMBEAM and Conclusion

Approximation of Deformations and Strains

(A) Small deformations and small strains are linear in coordinates of deformation $\mathbf{q}^i(t)$.

(B) Small deformations and small strains are linear in $\mathbf{q}^i(t)$ but considering of geometric stiffness matrices.

(C) Small strains are linear in $\mathbf{q}^i(t)$, but displacement field is quadratic in strain variables

(D) Linear displacement field approximation but strains are quadratic in displacement variables

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(D) Linear displacement field approximation but strains are quadratic in displacement variables

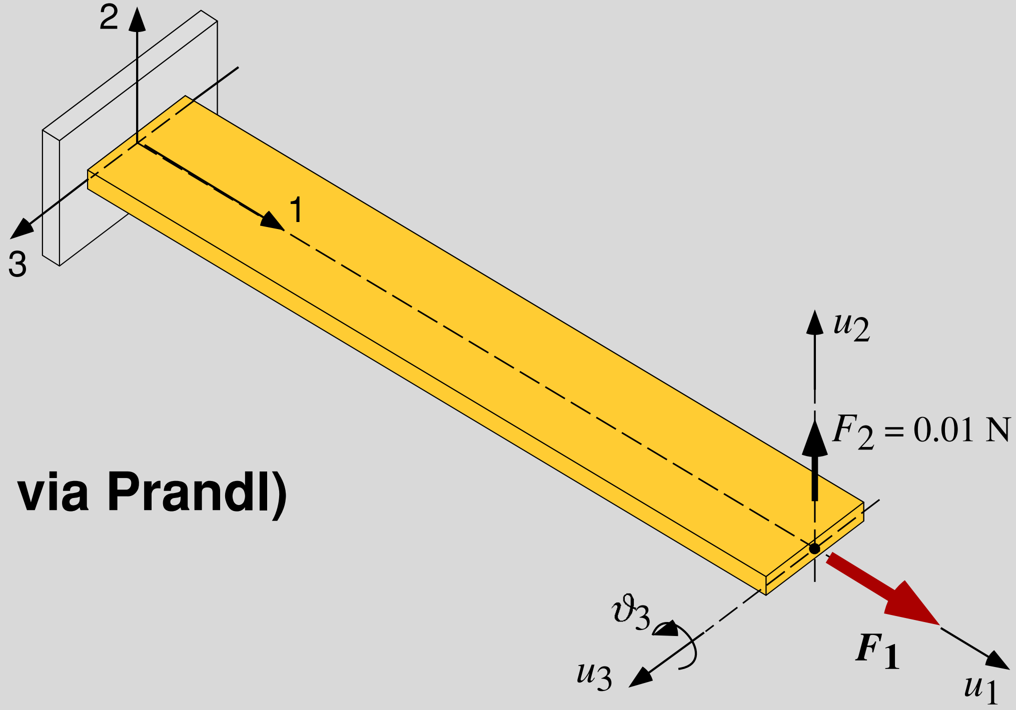
(E) Large deformations and large strains using FE-discretization; e.g. for beam structures (S.v.Dombrowski)

Approximation of Deformation and Strain

Model	Displacements	Internal Forces	Inertia Forces
A	linear: $\mathbf{u}^i = \mathbf{\Phi}^i \mathbf{q}^i$ $\boldsymbol{\vartheta}^i = \mathbf{\Psi}^i \mathbf{q}^i$	linear: $\delta\boldsymbol{\varepsilon}^i = \mathbf{B}^i \delta\mathbf{q}^i$ $\boldsymbol{\sigma}^i = \mathbf{H}^i \mathbf{B}^i \mathbf{q}^i$ $\mathbf{k}^i = \mathbf{K}_e^i \mathbf{q}^i$	$\mathbf{M}^i$ and $\mathbf{h}_\omega^i$ are quadratic in $\mathbf{q}^i$
B	linear: $\mathbf{u}^i = \mathbf{\Phi}^i \mathbf{q}^i$ $\boldsymbol{\vartheta}^i = \mathbf{\Psi}^i \mathbf{q}^i$	$\delta\varepsilon_\alpha^i = (\mathbf{B}_{L\alpha}^i + \mathbf{q}^{iT}\mathbf{B}_{Q\alpha}^i)\delta\mathbf{q}^i$ $\boldsymbol{\sigma}^i = \mathbf{H}^i \mathbf{B}^i \mathbf{q}^i + \boldsymbol{\sigma}_0^i$ $\mathbf{k}^i = (\mathbf{K}_e^i + \mathbf{K}_{geo}^i(\boldsymbol{\sigma}_0^i))\,\mathbf{q}^i$	$\mathbf{M}^i$ and $\mathbf{h}_\omega^i$ are quadratic in $\mathbf{q}^i$
C	quadr.: $\boldsymbol{\chi}^i = \mathbf{N}^i \mathbf{q}^i$ $u_\alpha^i = (\mathbf{\Phi}_{L\alpha}^i + \frac{1}{2}\mathbf{q}^{iT}\mathbf{\Phi}_{Q\alpha}^i)\,\mathbf{q}^i$ $\vartheta_\alpha^i = (\mathbf{\Psi}_{L\alpha}^i + \frac{1}{2}\mathbf{q}^{iT}\mathbf{\Psi}_{Q\alpha}^i)\,\mathbf{q}^i$	linear: $\delta\boldsymbol{\varepsilon}^i = \mathbf{B}^i \delta\mathbf{q}^i$ $\boldsymbol{\sigma}^i = \mathbf{H}^i \mathbf{B}^i \mathbf{q}^i$ $\mathbf{k}^i = \mathbf{K}_e^i \mathbf{q}^i$	$\mathbf{M}^i$ and $\mathbf{h}_\omega^i$ are function in $\mathbf{q}^i$ up to the fourth order
D	linear: $\mathbf{u}^i = \mathbf{\Phi}^i \mathbf{q}^i$ $\boldsymbol{\vartheta}^i = \mathbf{\Psi}^i \mathbf{q}^i$	quadr.: $\delta\varepsilon_\alpha^i = (\mathbf{B}_{L\alpha}^i + \mathbf{q}^{iT}\mathbf{B}_{Q\alpha}^i)\delta\mathbf{q}^i$ $\sigma_\alpha^i = H_{\alpha\alpha}^i(\mathbf{B}_{L\alpha}^i + \frac{1}{2}\mathbf{q}^{iT}\mathbf{B}_{Q\alpha}^i)\mathbf{q}^i\,\mathbf{k}^i$ $\mathbf{k}^i(\mathbf{q}^i)$ is cubic in $\mathbf{q}^i$	$\mathbf{M}^i$ and $\mathbf{h}_\omega^i$ are quadratic in $\mathbf{q}^i$

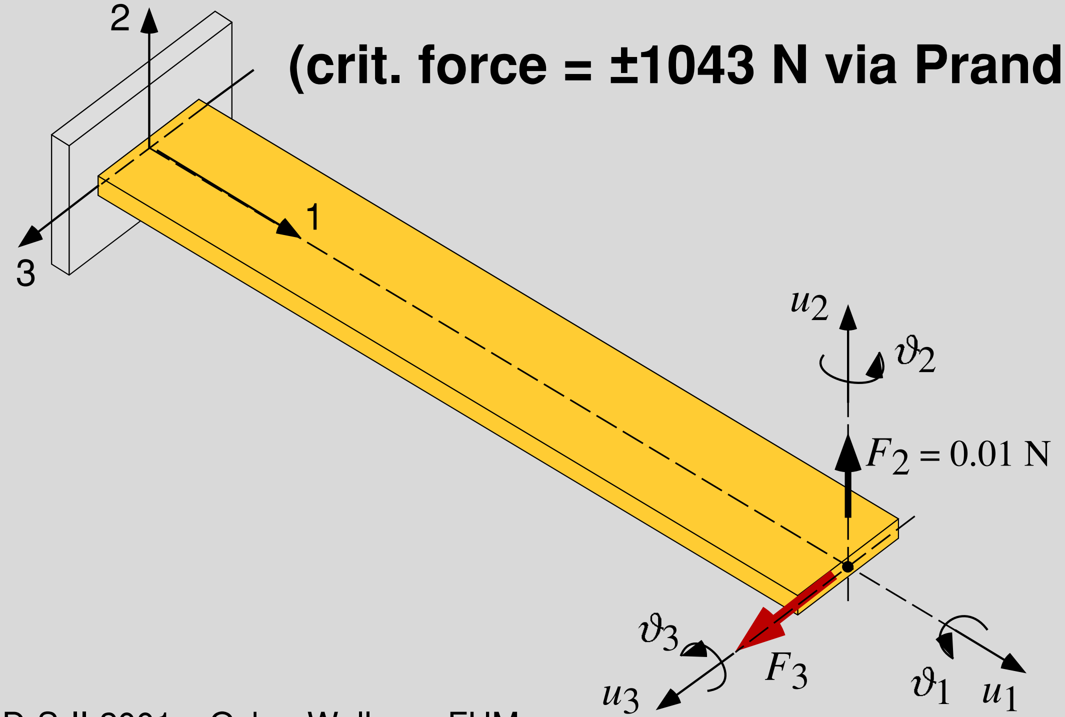
Examples to Test the Approximations

Load case 1 – Buckling (crit. force = –525 N via Euler)



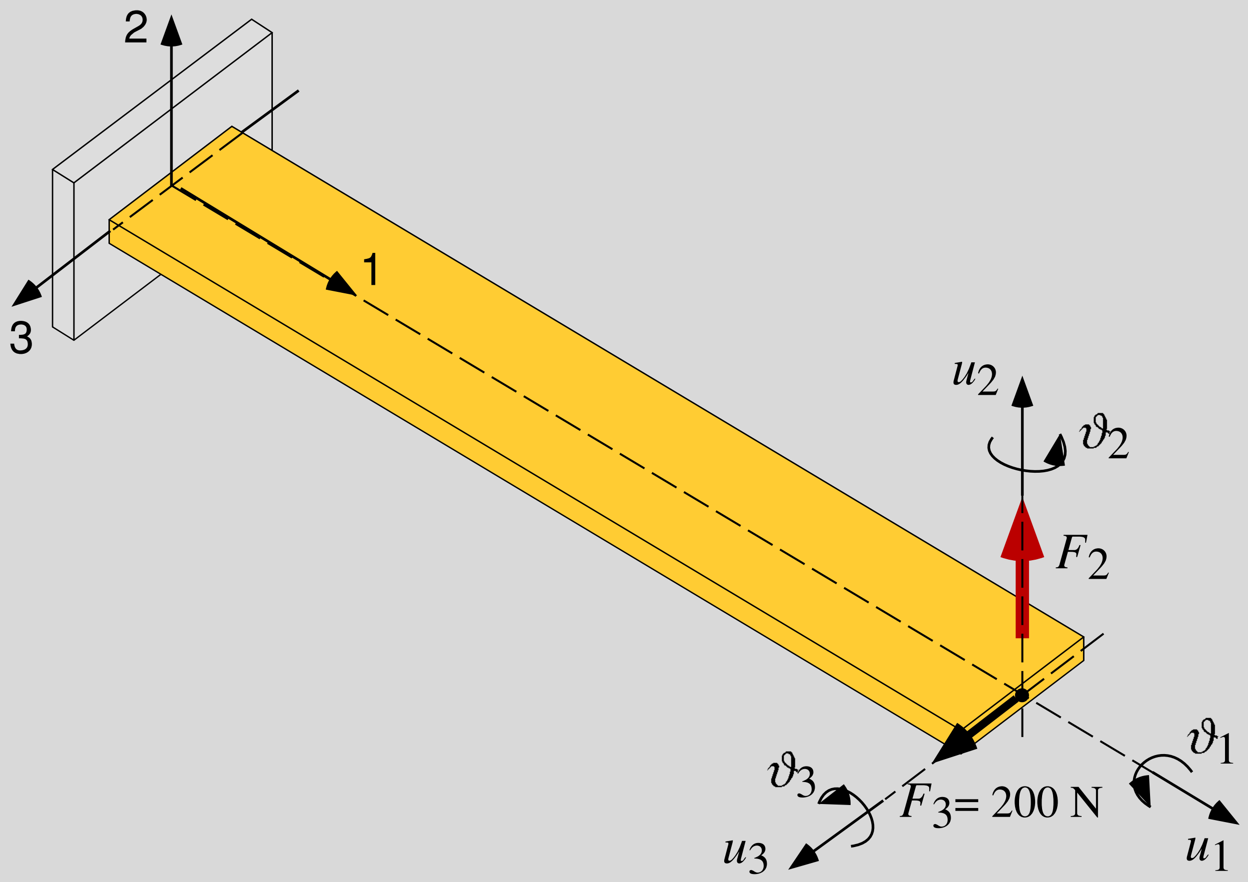
Load case 2 – Tilting

(crit. force = ±1043 N via Prandl)



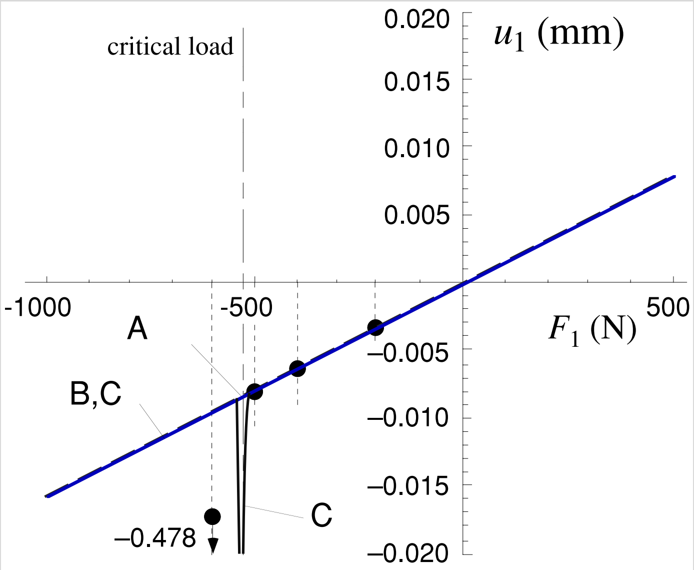
Examples to Test the Approximations

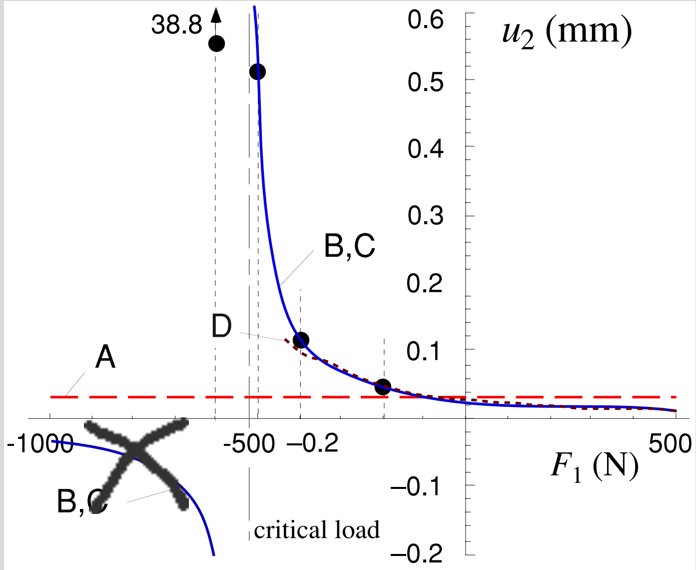
Load case 3 – Bending in soft direction (quadratic effects for u_1, u_3, ϑ_2)



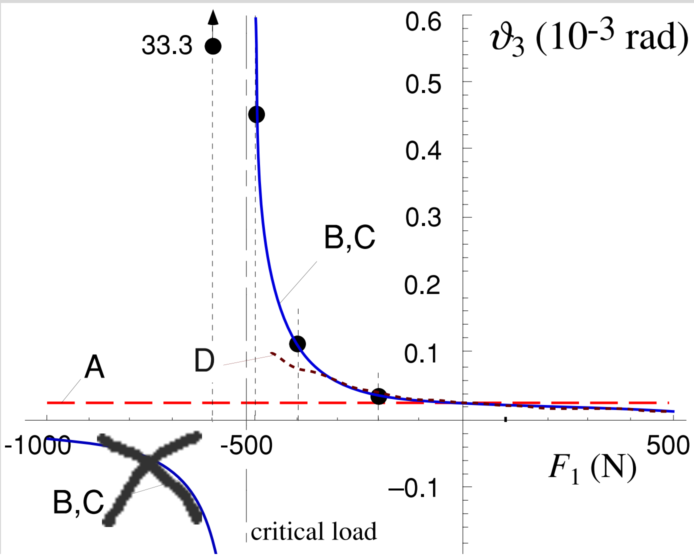
Solution of the Test Examples

Load case 1 – Buckling (crit. force = –525 N via Euler)





• = (E)



Results:

(A) u_2, ϑ_3 independent of F_1, no buckling

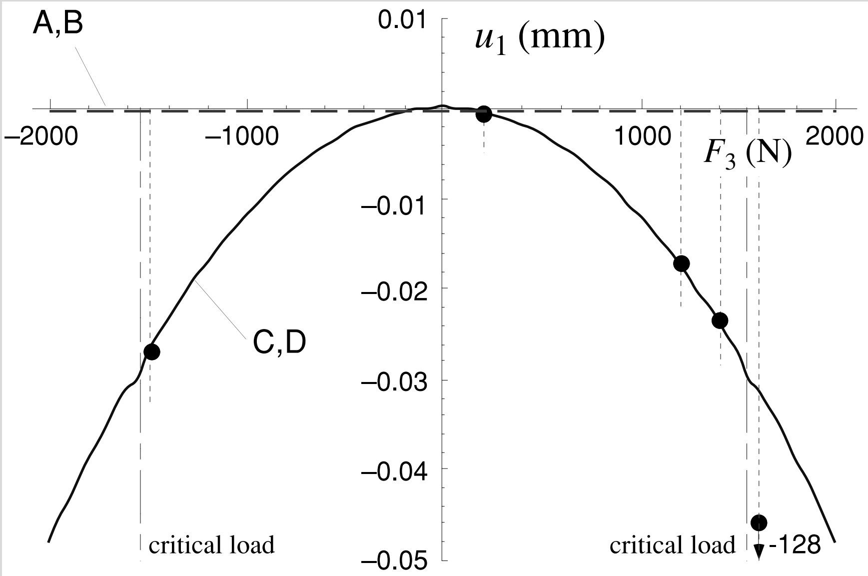
(B),(C) displacements are correct for $F_1 > F_{crit}$

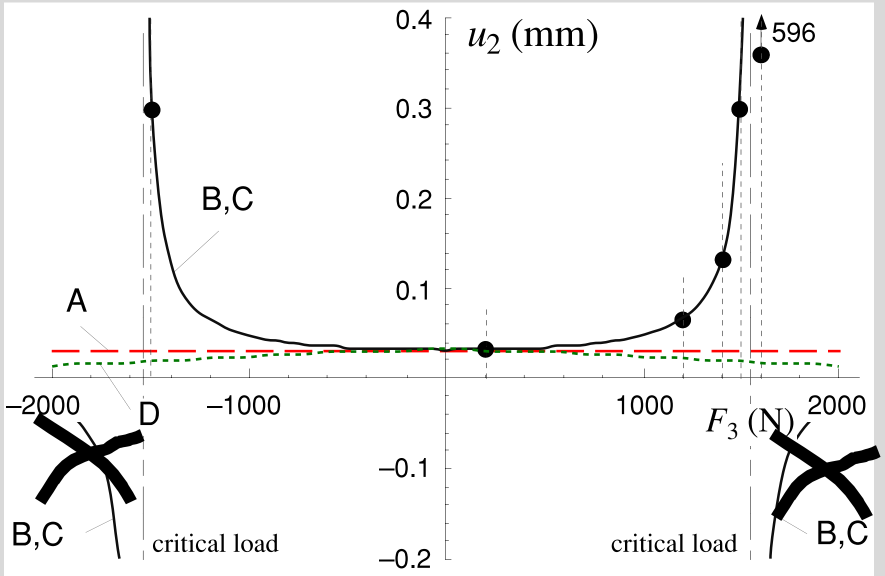
(D) solution failed for F_1,< 1/2 F_{crit}

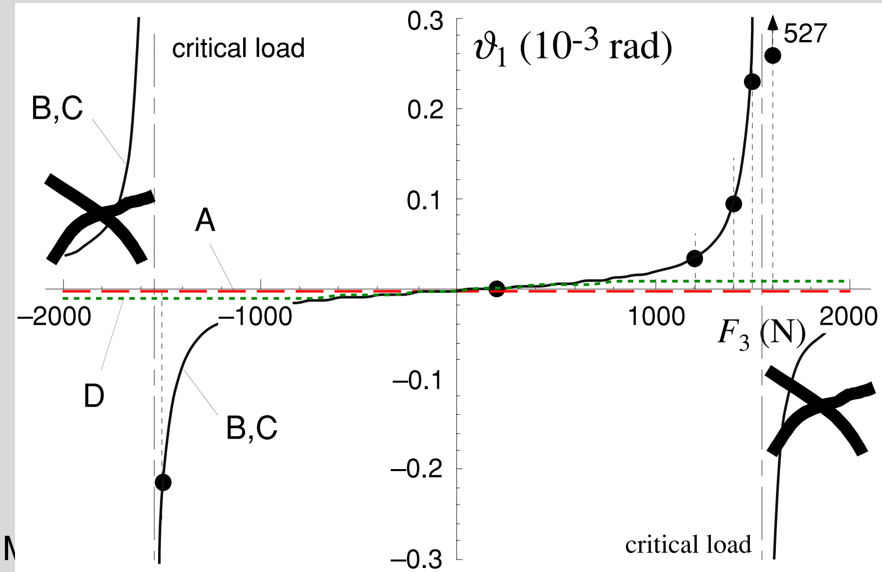
Solution of the Test Examples

• = (E)

Load case 2 – Tilting (crit. force = ±1043 N via Prandl, 1560 N for 1 El.)







Results:

(A) u_1, u_2, ϑ_1, ϑ_3 independent of F_3, no tilting

(B) u_1 independent of F_3, no shortening

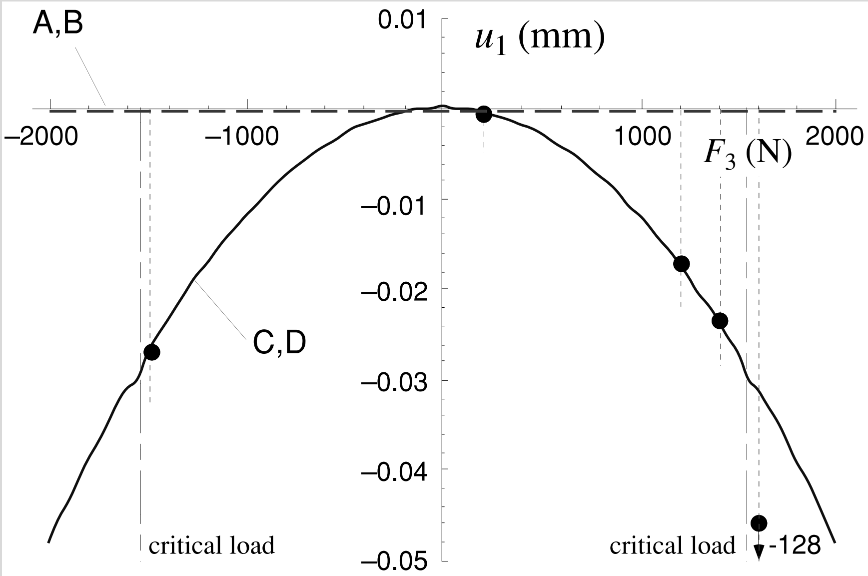
(B),(C) displacements correct (< 1 %) for $|F_3| < F_{crit}$

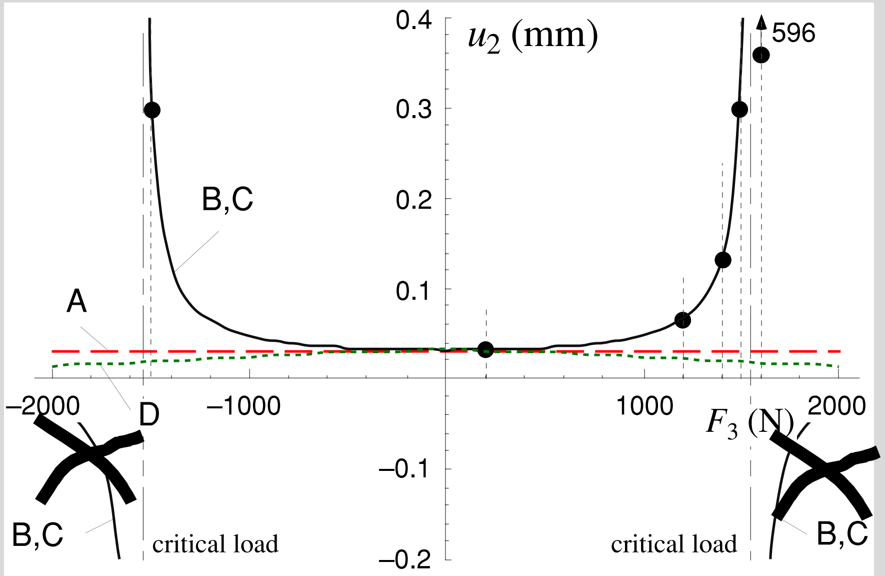
(D) solution failed

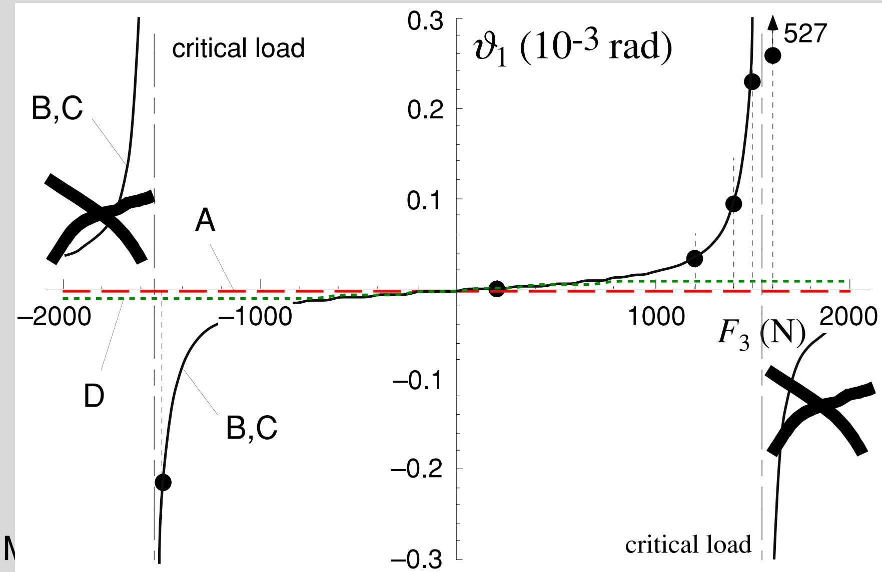
Solution of the Test Examples

• = (E)

Load case 2 – Tilting (crit. force = ±1043 N via Prandl, 1560 N for 1 El.)







Results:

(A) u_1, u_2, ϑ_1, ϑ_3 independent of F_3, no tilting

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(D) solution failed

Design of SIMBEAM

1. **For 3D-beam structures**
   model (B) considers buckling and tilting
   (B) = linear displacement approximation + geometric stiffening matrices

2. **Quadratic effects of shortening and bending coupling (u_1, u_3, ϑ_2) are considered by displacement corrections due to a quadratic fcts. in $\mathbf{q}^i(t)$ as given in model (C) :**

$$u^i_\alpha(x,t) = \mathbf{\Phi}^i_{L\alpha}(x)\,\mathbf{q}^i + \frac{1}{2}\mathbf{q}^{iT}\,\mathbf{\Phi}^i_{Q\alpha}(x)\,\mathbf{q}^i$$

3. **Matrices $\mathbf{\Phi}^i_{Q\alpha}$ are derived from quadric strain-displacement relation and pre-computed in SIMBEAM.**

4. **For data transfer, SIMPACK uses SID technology.**

⇨ **SIMBEAM uses items 1, 2, 3, 4**

Conclusion

For flexible MBS using floating frame of reference

1. **a linear approximation of displacement field**

   $$\mathbf{u}(\mathbf{R},t) = \mathbf{\Phi}_L(\mathbf{R})\ \ \mathbf{q}(t)$$

   by incorporating geometric stiffening effects allows to analyze buckling and tilting problems

   but !
   never use results for loads higher than the critical load

2. **shortening and bending coupling will be appear by quadratic correction terms** $+\ 1/2\ \ \mathbf{q}^{\mathrm{T}}\ \mathbf{\Phi}_{Q\alpha}\,\mathbf{q}$ **of displacement field**

3. **the errors of results are in an acceptable range**

Conclusion

SIMBEAM is a module of SIMPACK

4. **satisfying the above items of approximation**
5. **prepare the coefficients of system matrices of a flexible body**
6. **the body is a 3D-FE-net of beam elements**
7. **useful for nodal and modal approach**

Thank you for your attention !