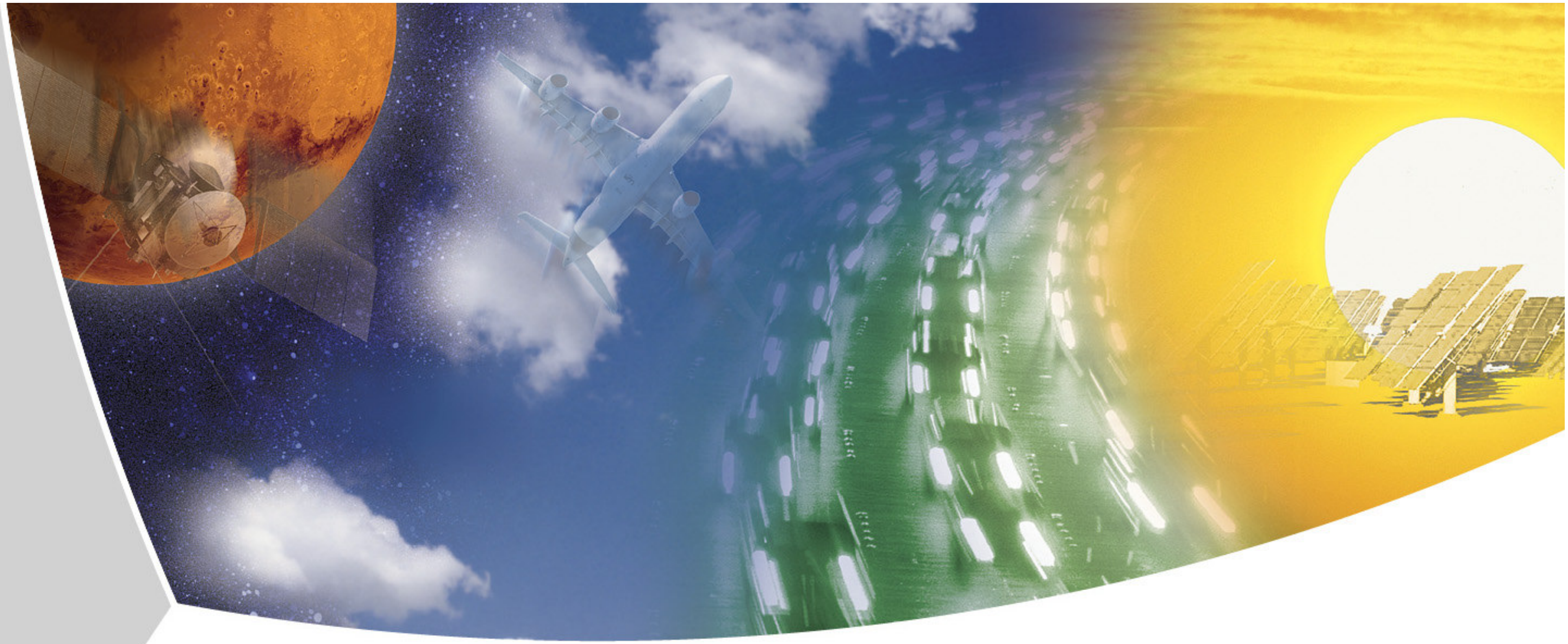

Multi-body Dynamics for Aeroelastic Stability Analysis of a Helicopter in Forward Flight

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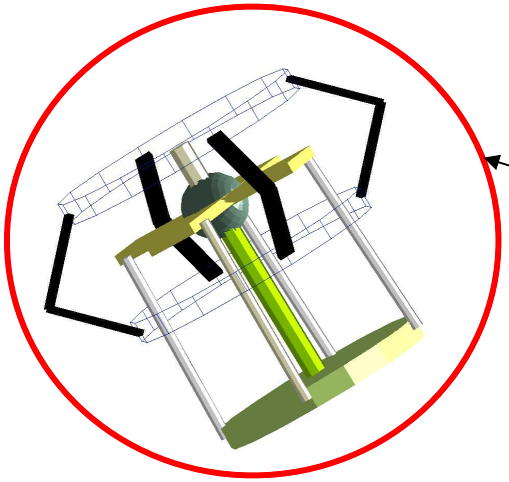
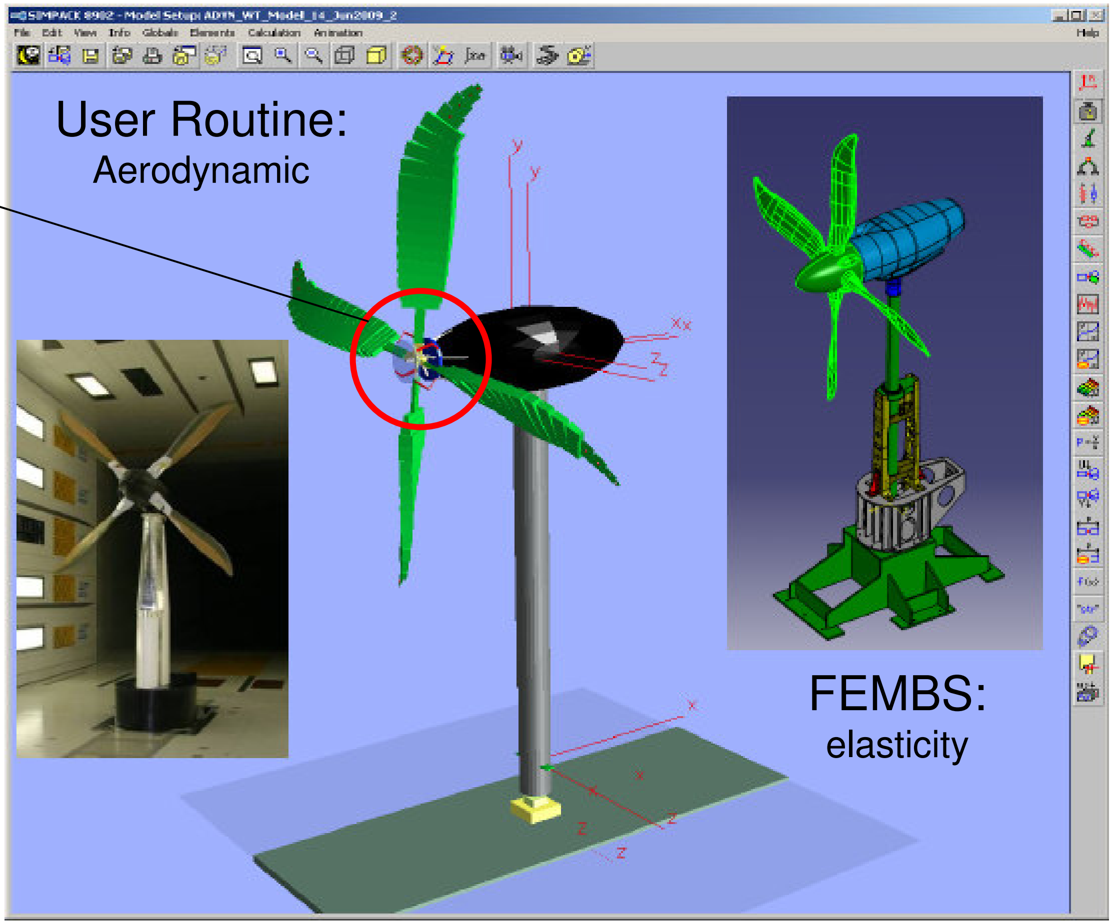
DLR Deutsches Zentrum für Luft- und Raumfahrt e.V.
in der Helmholtz-Gemeinschaft

Content

Simulation Experience with SIMPACK



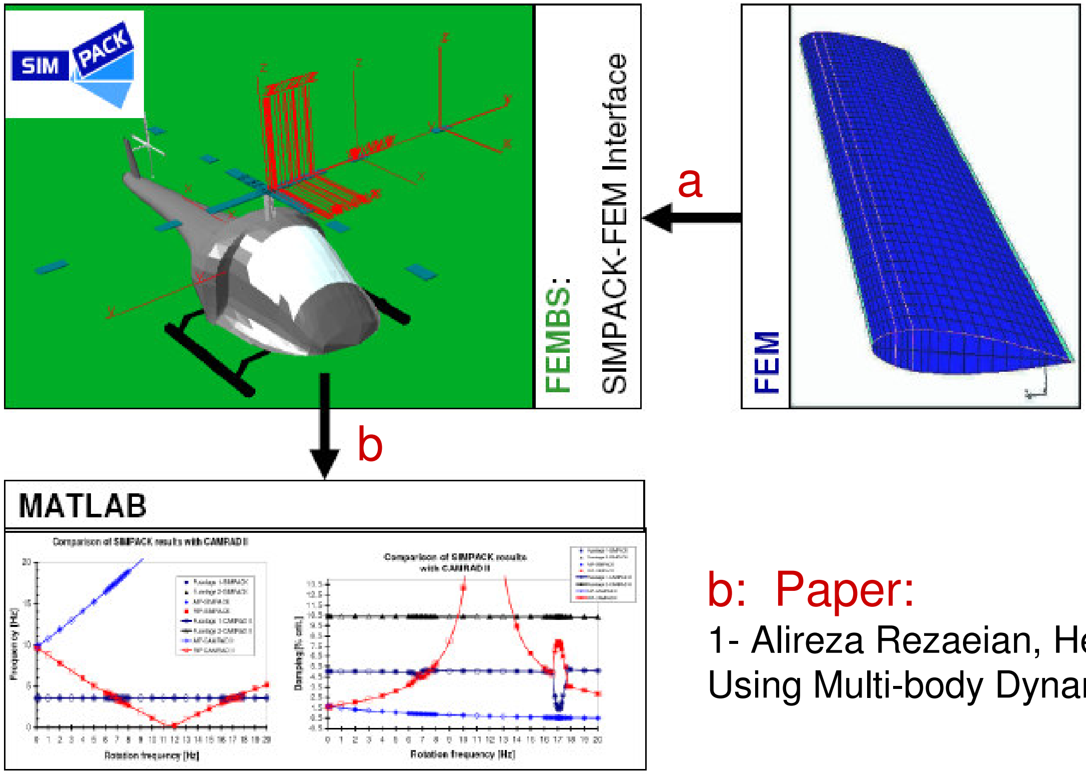


User Routine:
Aerodynamic

FEMBS:
elasticity

Paper: Dr. Wolf R. Krüger, Multi-body Analysis of Whirl Flutter Dynamics on a Tiltrotor Wind Tunnel Model, International Forum of Aeroelasticity and Structural Dynamics (IFASD) 2009

Simulation Experience with SIMPACK



a: Paper:

1- Dr. Stefan Waitz, From FEM to MBS: Stability Analysis of the Elastic H/C-Rotor, European Rotorcraft Forum (ERF) 2010

2- Jürgen Arnold, Using Multi-body Dynamics for the Simulation of Flexible Rotor Blades- Modelling Limits of an Innovative Blade Layout based on Beam Approach, ERF 2010

b: Paper:

1- Alireza Rezaeian, Helicopter Ground Resonance Analysis Using Multi-body Dynamics, European Rotorcraft Forum 2010

Stability Analysis of a System with Linear Differential Equation

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{0\}$$

$$\Downarrow$$

$$\{\ddot{q}\}+[M]^{-1}[C]\{\dot{q}\}+[M]^{-1}[K]\{q\}=\{0\}$$

$$\Downarrow$$

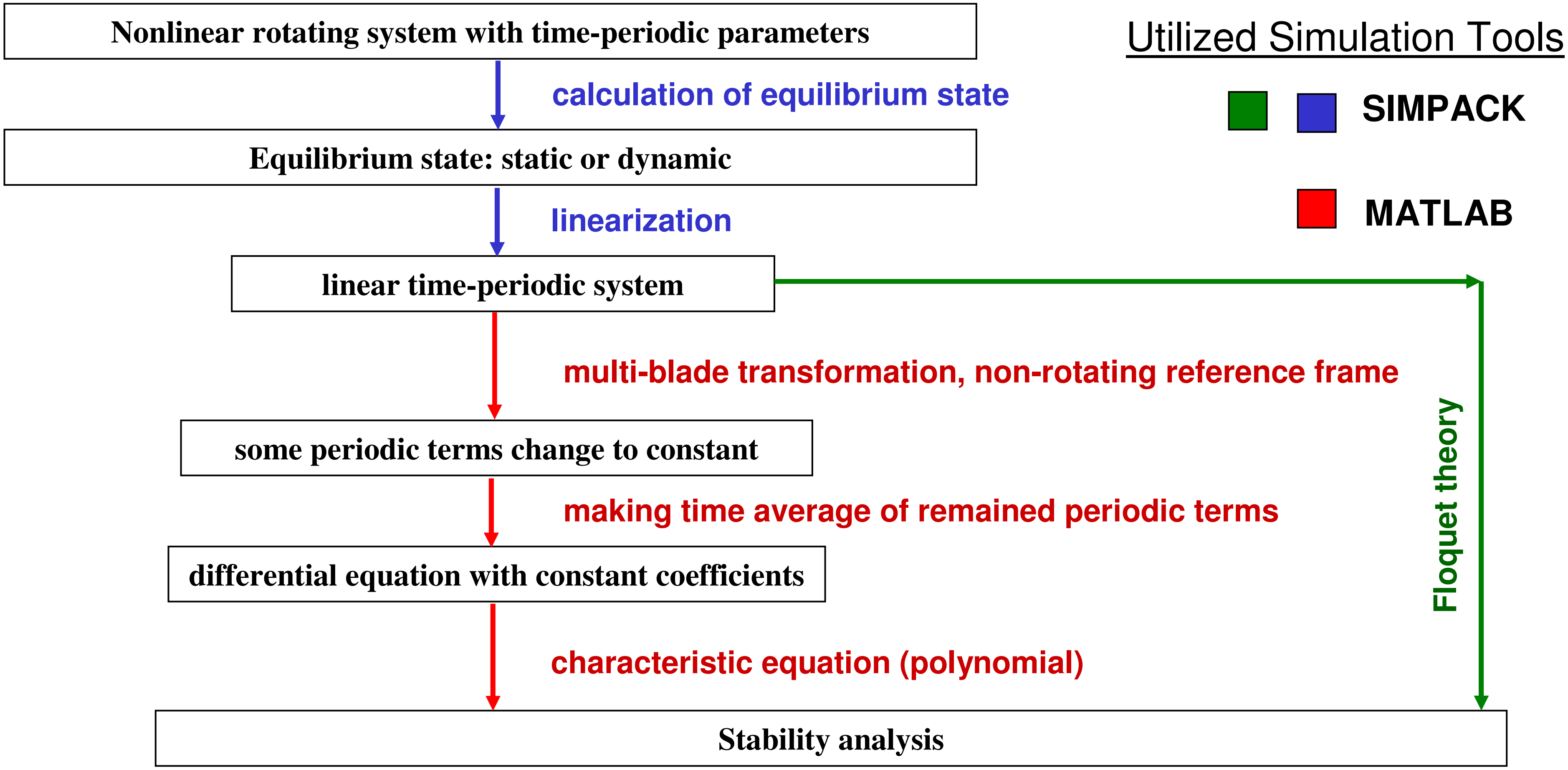
$$\begin{Bmatrix}\{\dot{q}\}\\ \cdots \\ \{\ddot{q}\}\end{Bmatrix}=\begin{bmatrix}0 & \vdots & [I]\\ \cdots & \vdots & \cdots\\ -[M]^{-1}[K] & \vdots & -[M]^{-1}[C]\end{bmatrix}\begin{Bmatrix}\{q\}\\ \cdots \\ \{\dot{q}\}\end{Bmatrix} \xrightarrow{\{X\}=\begin{Bmatrix}\{q\}\\ \cdots \\ \{\dot{q}\}\end{Bmatrix}} \{\dot{X}\}=\underset{}{[A]}\{X\}$$

linear system matrix

1- Elements of „A“ are constant in time → Eigenvalue → stability analysis

2- Elements of „A“ are time-periodic → Classical eigenvalue analysis is not valid

Stability Analysis of non-linear Time-periodic Systems



Nonlinear rotating system with time-periodic parameters

calculation of equilibrium state

Equilibrium state: static or dynamic

linearization

linear time-periodic system

multi-blade transformation, non-rotating reference frame

some periodic terms change to constant

making time average of remained periodic terms

differential equation with constant coefficients

characteristic equation (polynomial)

Floquet theory

Stability analysis

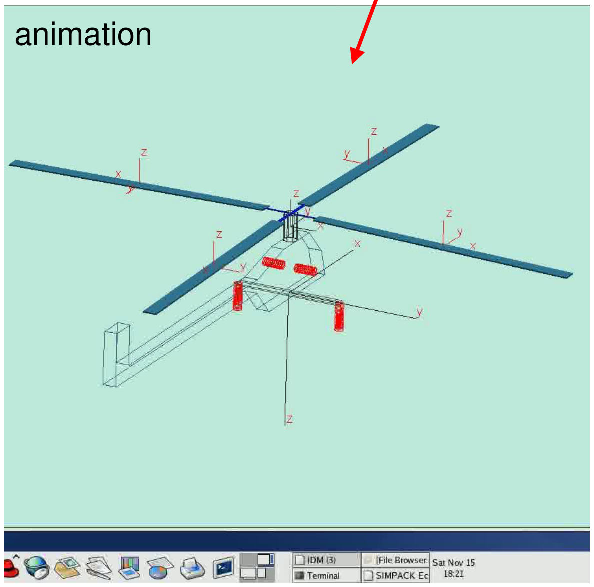
Utilized Simulation Tools

🟩 🟦 **SIMPACK**

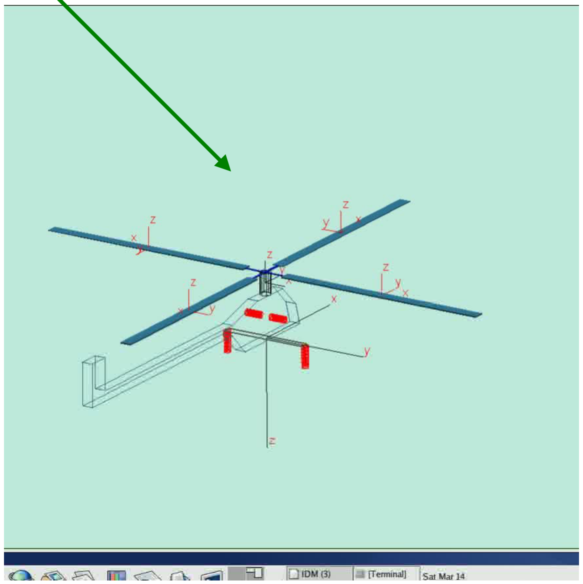
🟥 **MATLAB**

Multi-blade Coordinates (eg. Flapping)

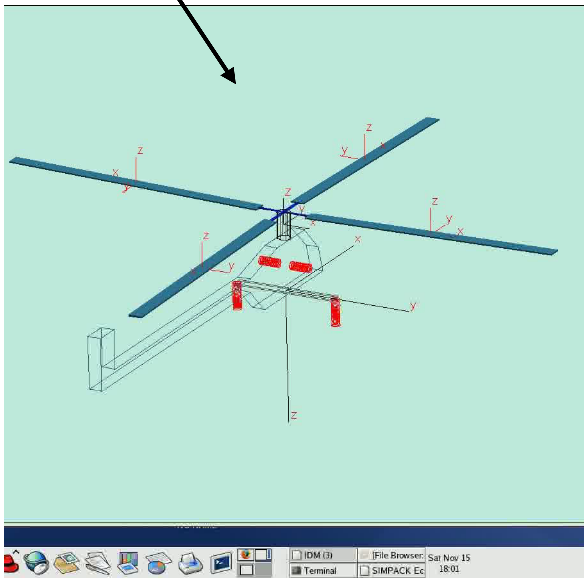
$$\beta_i = \beta_0 + \beta_{1C} \cos\psi_i + \beta_{1S} \sin\psi_i + \beta_d (-1)^i$$



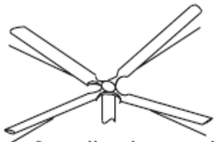
rotor coning angle



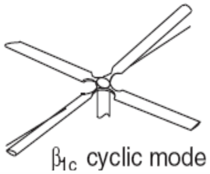
longitudinal flapping



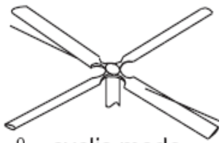
differential flapping



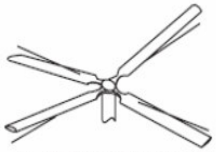
β_0 collective mode



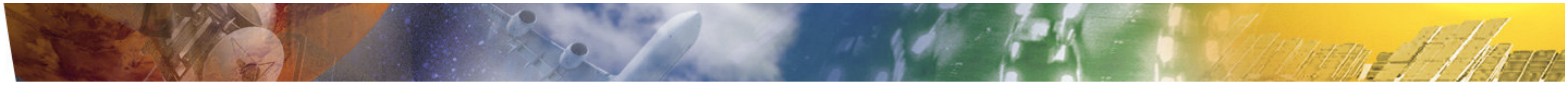
β_{1c} cyclic mode



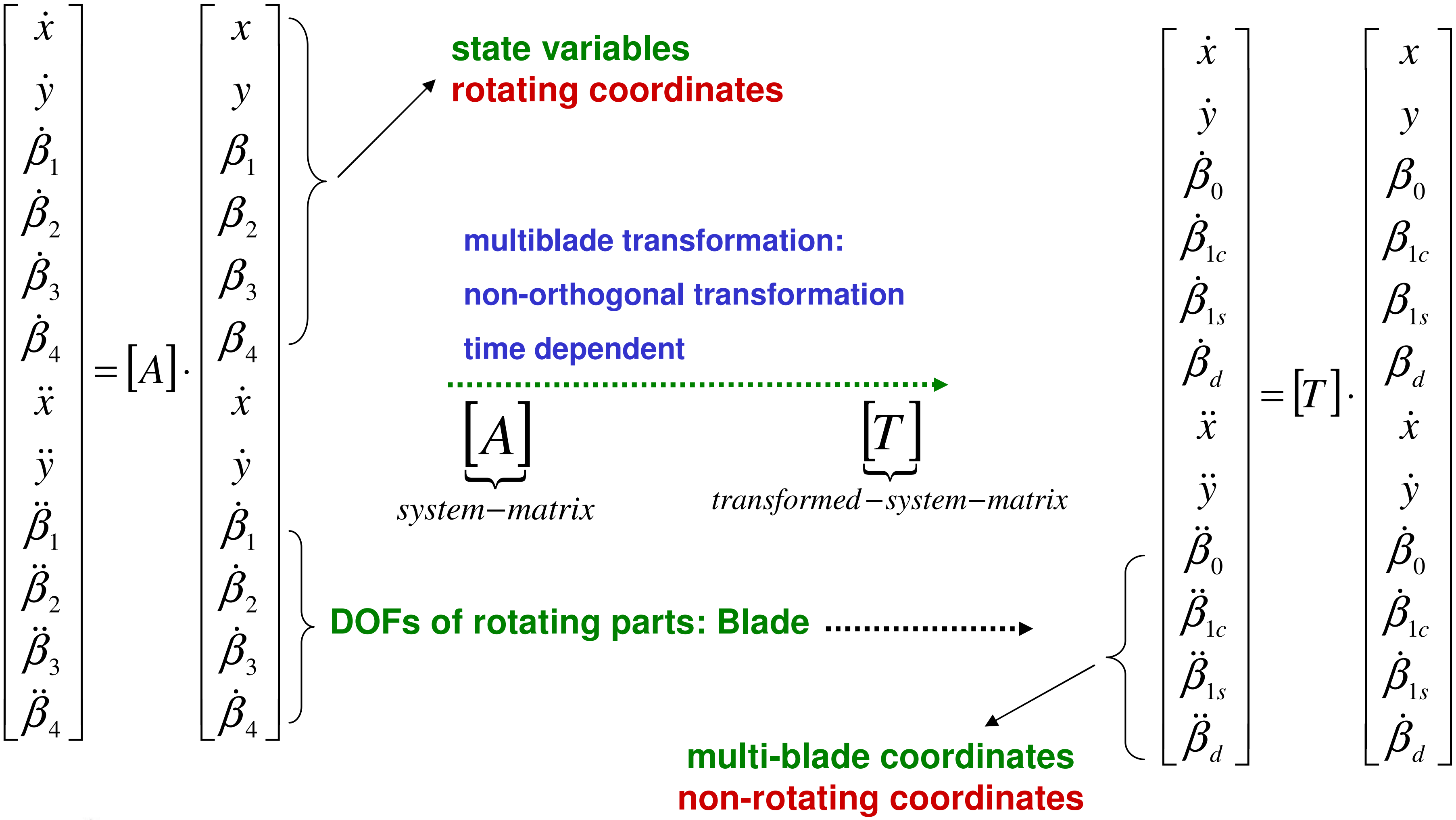
β_{1s} cyclic mode



β_d differential mode



Multi-blade Coordinates Transformation



$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\beta}_1\\ \dot{\beta}_2\\ \dot{\beta}_3\\ \dot{\beta}_4\\ \ddot{x}\\ \ddot{y}\\ \ddot{\beta}_1\\ \ddot{\beta}_2\\ \ddot{\beta}_3\\ \ddot{\beta}_4\end{bmatrix} = [A]\cdot\begin{bmatrix}x\\ y\\ \beta_1\\ \beta_2\\ \beta_3\\ \beta_4\\ \dot{x}\\ \dot{y}\\ \dot{\beta}_1\\ \dot{\beta}_2\\ \dot{\beta}_3\\ \dot{\beta}_4\end{bmatrix}$$

state variables

rotating coordinates

multiblade transformation:

non-orthogonal transformation

time dependent

$$\underbrace{[A]}_{system-matrix} \qquad \underbrace{[T]}_{transformed-system-matrix}$$

DOFs of rotating parts: Blade

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\beta}_0\\ \dot{\beta}_{1c}\\ \dot{\beta}_{1s}\\ \dot{\beta}_d\\ \ddot{x}\\ \ddot{y}\\ \ddot{\beta}_0\\ \ddot{\beta}_{1c}\\ \ddot{\beta}_{1s}\\ \ddot{\beta}_d\end{bmatrix} = [T]\cdot\begin{bmatrix}x\\ y\\ \beta_0\\ \beta_{1c}\\ \beta_{1s}\\ \beta_d\\ \dot{x}\\ \dot{y}\\ \dot{\beta}_0\\ \dot{\beta}_{1c}\\ \dot{\beta}_{1s}\\ \dot{\beta}_d\end{bmatrix}$$

multi-blade coordinates

non-rotating coordinates

Content

Fundament: Simulation Experience with SIMPACK, Stability Analysis, Multi-blade Coordinates, Multi-blade Coordinates Transformation

Rotor Blade Flapping Motion in Forward Flight

Flapping Stability of Helicopter Rotor in Forward Flight

- Method of Analysis
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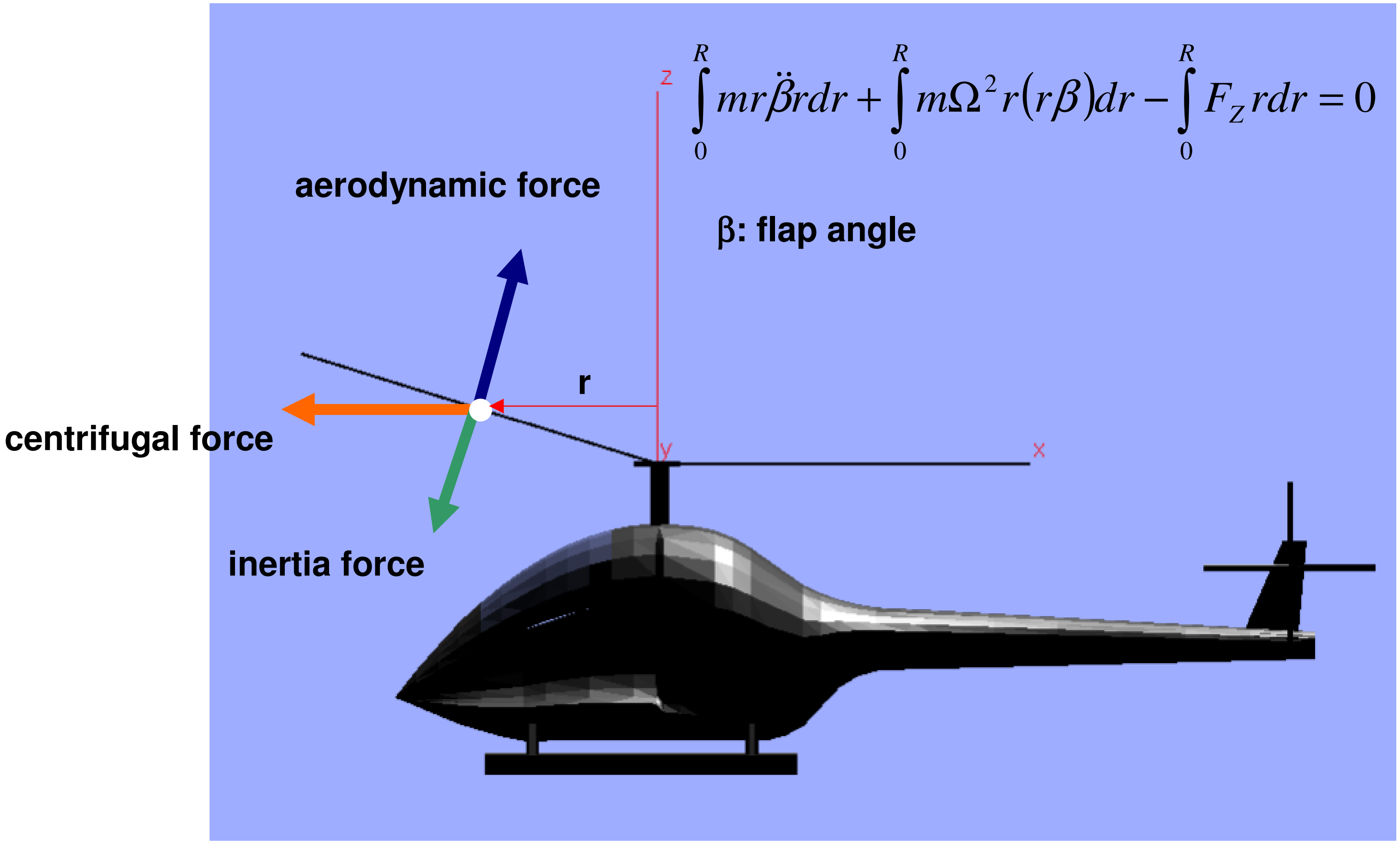
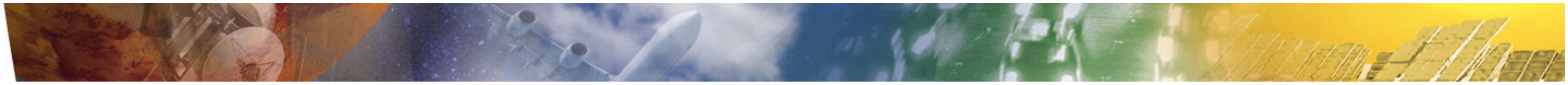
Equation of Flapping Motion of a Rigid Rotor Blade in Forward Flight



$$\int_0^R mr\ddot{\beta}rdr + \int_0^R m\Omega^2 r(r\beta)dr - \int_0^R F_Z rdr = 0$$

β: flap angle



Equation of Flapping Motion of a Rigid Rotor Blade in Forward Flight

Blade: rigid, spring restrained, centrally hinged blade, **without reverse flow**

B: tip loss factor

Kβ: flapping spring stiffness

$$\ddot{\beta}_i + \left(\frac{\gamma}{8}B^4 + \mu\frac{\gamma}{6}B^3\sin\psi_i\right)\dot{\beta}_i + \left(1+\frac{k_\beta}{I\Omega^2}+\mu\frac{\gamma}{6}B^3\cos\psi_i+\mu^2\frac{\gamma}{8}B^2\sin 2\psi_i\right)\beta_i$$

μ: rotor advance ratio

I: blade flapping inertia

ψ: azimuth angle

γ: blade lock number

$$\gamma = \frac{\rho a c R^4}{I}$$

a: blade lift curve slope

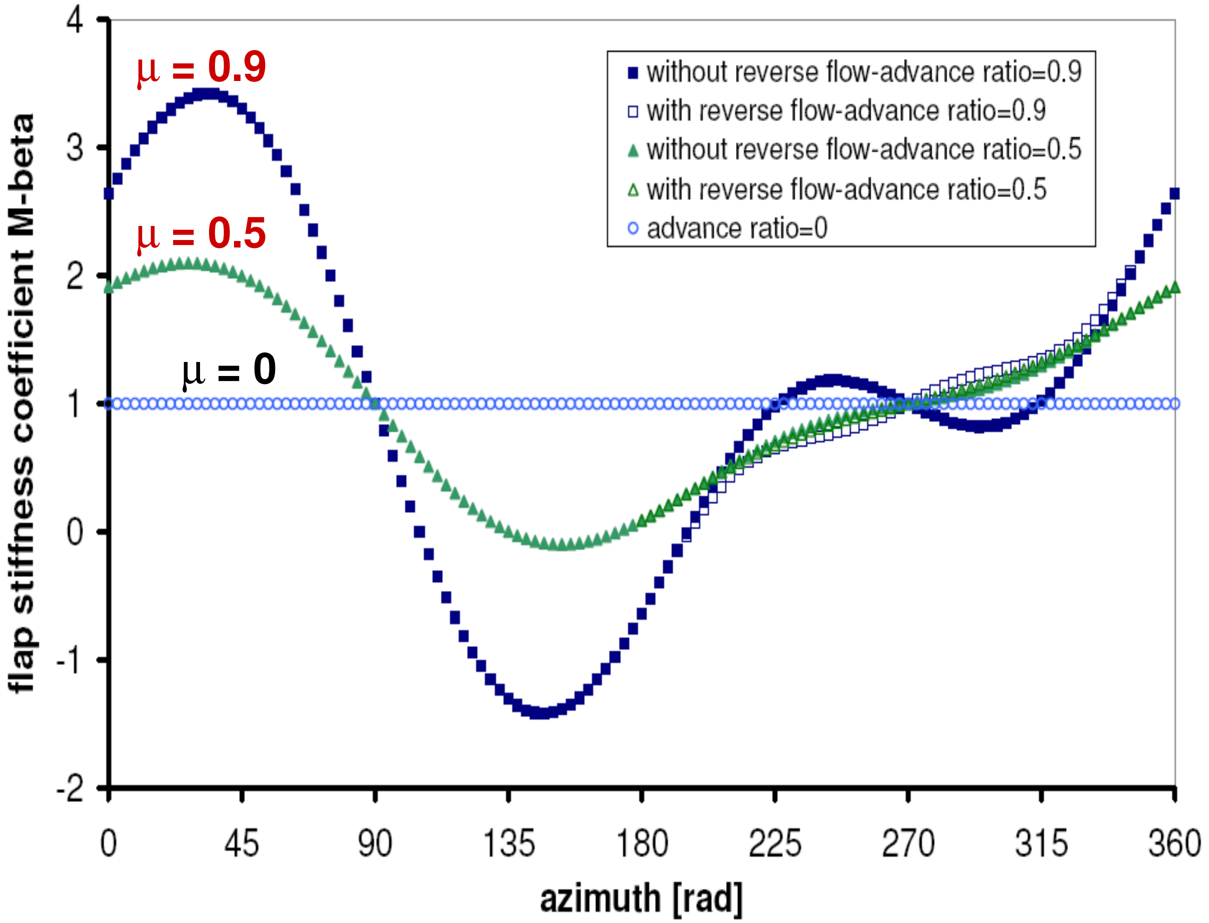
Equation of Flapping Motion of a Rigid Rotor Blade in **Forward Flight**

Blade: rigid, spring restrained, centrally hinged blade, **with reverse flow**

$$180-(i-1)\frac{\pi}{2} < \psi_i < 360-(i-1)\frac{\pi}{2}$$

$$\ddot{\beta}_i + \left(\underbrace{\frac{\gamma}{8}B^4 + \mu\frac{\gamma}{6}B^3\sin\psi_i + \frac{\gamma}{12}\mu^4\sin^4\psi_i}_{\boxed{\text{M-beta-dot}}}\right)\dot{\beta}_i + \left(\underbrace{1+\frac{k_\beta}{I\Omega^2} + \mu\frac{\gamma}{6}B^3\cos\psi_i + \mu^2\frac{\gamma}{8}B^2\sin 2\psi_i - \frac{\gamma}{6}\mu^4\cos\psi_i\sin^3\psi_i}_{\boxed{\text{M-beta}}}\right)\beta_i$$

Stiffness Coefficient of the Flapping Motion Equation with and without Reverse Flow

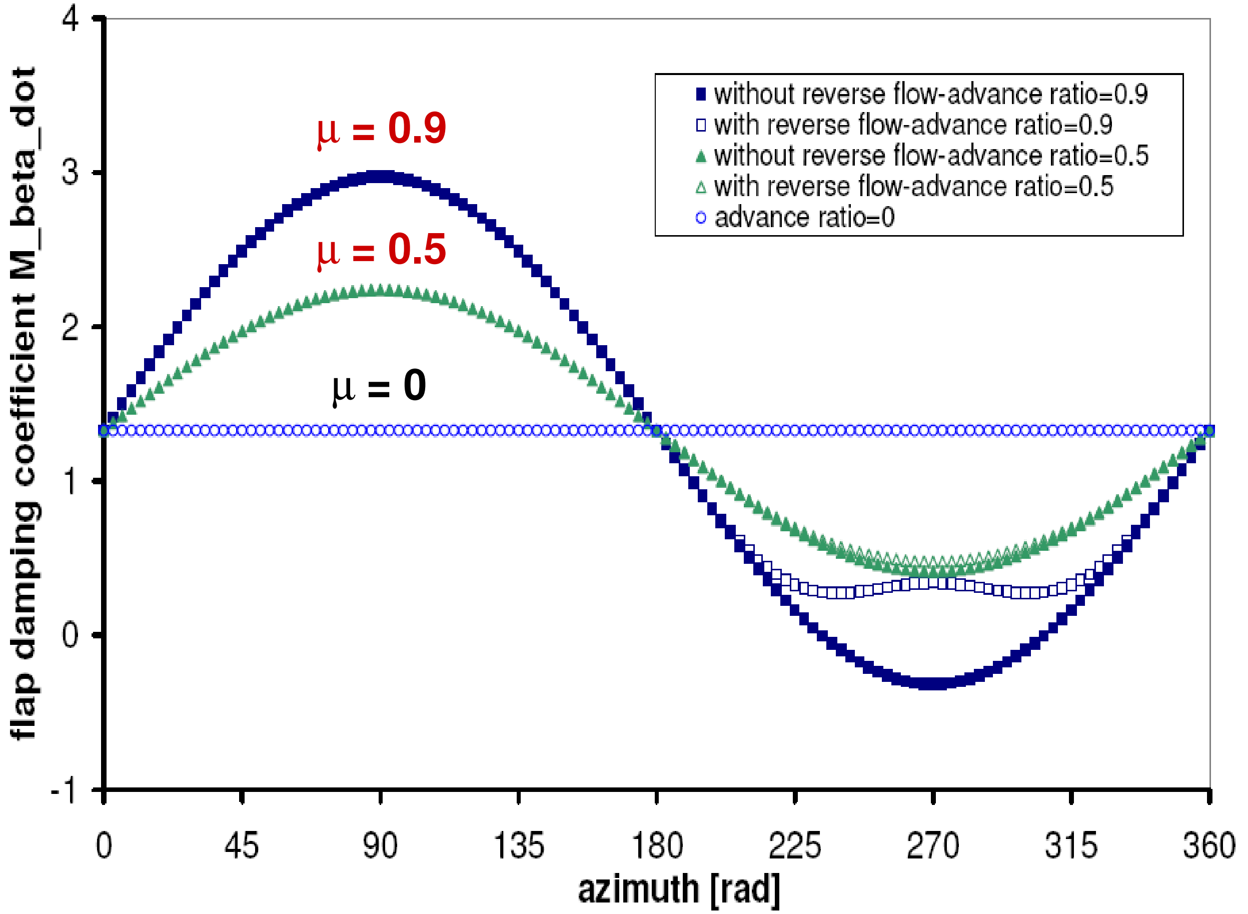


B = 0.97

$\gamma = 12$

$K\beta = 0$

Damping Coefficient of the Flapping Motion Equation with and without Reverse Flow



B = 0.97

$\gamma = 12$

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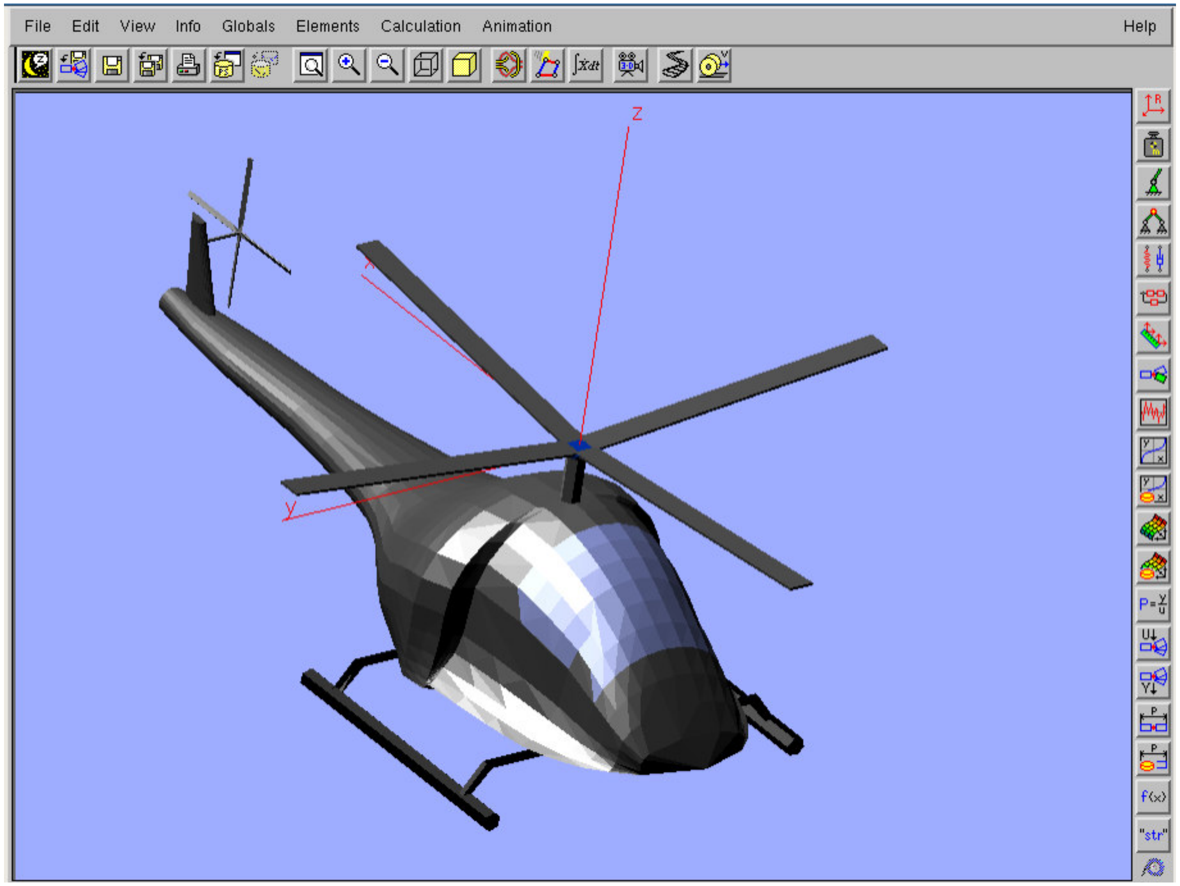
Flapping Stability of Helicopter Rotor in Forward Flight: Method of Analysis



SIMPACK

1- Equilibrium state

2- Linearization about the equilibrium state

3- Obtaining the linear system matrices for the different azimuth angles

MATLAB as postprocessor

4- Multi-blade transformation of the linear system matrices and then calculation of the time average matrix

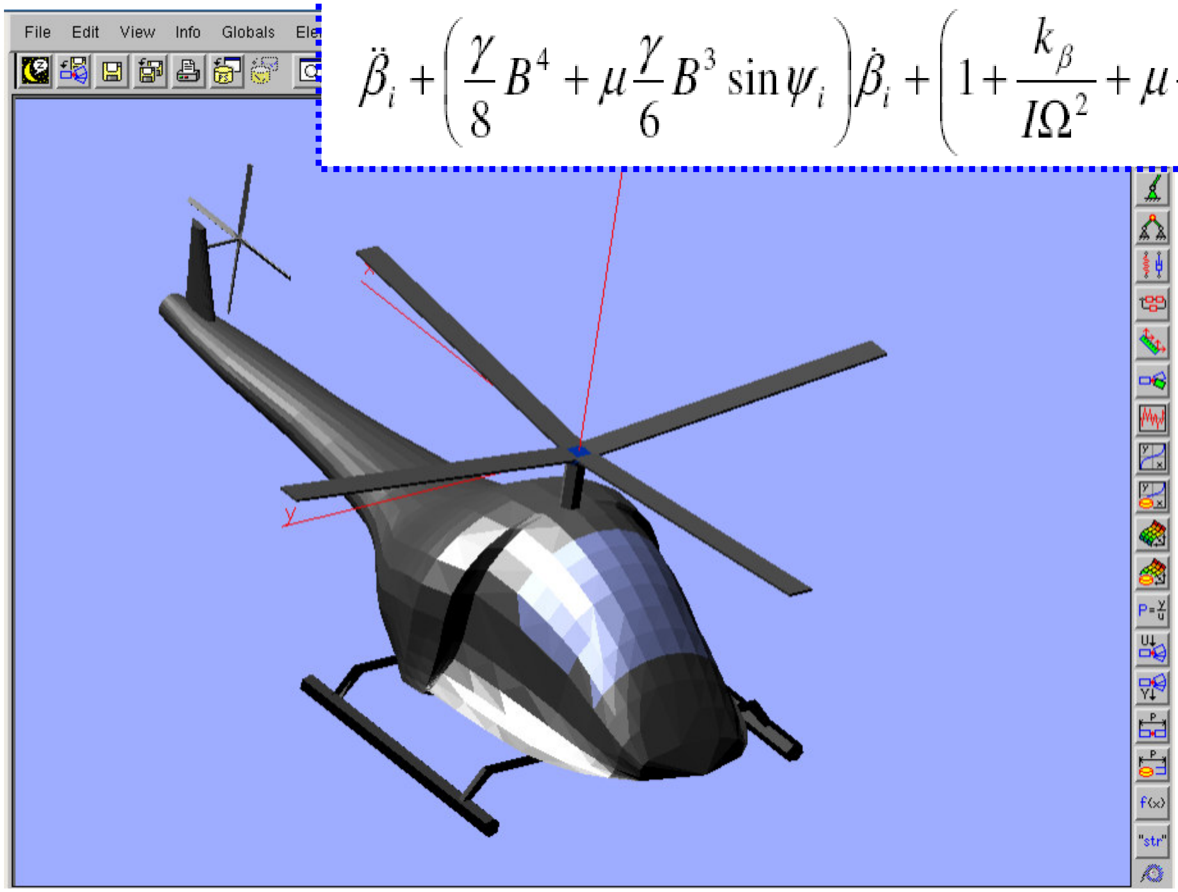
5- Eigenvalues of the calculated time averaged matrix

6- Sorting the eigenforms

7- Plotting the results

Flapping Stability of Helicopter Rotor in Forward Flight: SIMPACK Model

$$\ddot{\beta}_i + \left(\frac{\gamma}{8}B^4 + \mu\frac{\gamma}{6}B^3\sin\psi_i\right)\dot{\beta}_i + \left(1 + \frac{k_\beta}{I\Omega^2} + \mu\frac{\gamma}{6}B^3\cos\psi_i + \mu^2\frac{\gamma}{8}B^2\sin 2\psi_i\right)\beta_i$$



Model parameters

$$\left(B, \gamma, \mu, k_\beta, \Omega\right)$$

↓

SIMPACK Expressions

↓

SIMPACK Force Elements

Flapping Stability of Helicopter Rotor in Forward Flight: Results and Validation

SIMPACK Linear System Matrix

System Matrix "A" from SIMPACK:

$\Omega = 1,\ B = 0.97\ ,\ \mu = 0.3,\ \gamma = 12,\ k_\beta = 0, \psi = 67.5$

```
A =

          0         0         0         0    1.0000         0         0         0
          0         0         0         0         0    1.0000         0         0
          0         0         0         0         0         0    1.0000         0
          0         0         0         0         0         0         0    1.0000
    -1.2994         0         0         0   -1.8339         0         0         0
          0   -0.4043         0         0         0   -1.5375         0         0
          0         0   -0.8803         0         0         0   -0.8220         0
          0         0         0   -1.4161         0         0         0   -1.1184
```

Comparing the SIMPACK result with analytical calculations:

$$a_{51} = -1 - \frac{k_\beta}{I\Omega^2} - \mu\frac{\gamma}{6}B^3\cos\psi_i - \mu^2\frac{\gamma}{8}B^2\sin 2\psi_i = -1.2994$$

$$a_{55} = -\frac{\gamma}{8}B^4 - \mu\frac{\gamma}{6}B^3\sin\psi_i = -1.8339$$

$$a_{62} = -1 - \frac{k_\beta}{I\Omega^2} + \mu\frac{\gamma}{6}B^3\sin\psi_i + \mu^2\frac{\gamma}{8}B^2\sin 2\psi_i = -0.4043$$

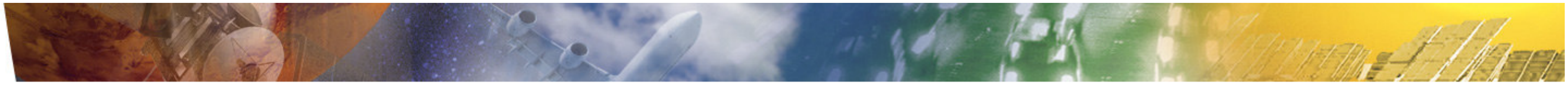
$$a_{66} = -\frac{\gamma}{8}B^4 - \mu\frac{\gamma}{6}B^3\cos\psi_i = -1.5375$$

$$a_{73} = -1 - \frac{k_\beta}{I\Omega^2} + \mu\frac{\gamma}{6}B^3\cos\psi_i - \mu^2\frac{\gamma}{8}B^2\sin 2\psi_i = -0.8803$$

$$a_{77} = -\frac{\gamma}{8}B^4 + \mu\frac{\gamma}{6}B^3\sin\psi_i = -0.8220$$

$$a_{84} = -1 - \frac{k_\beta}{I\Omega^2} - \mu\frac{\gamma}{6}B^3\sin\psi_i + \mu^2\frac{\gamma}{8}B^2\sin 2\psi_i = -1.4161$$

$$a_{88} = -\frac{\gamma}{8}B^4 + \mu\frac{\gamma}{6}B^3\cos\psi_i = -1.1184$$



Flapping Stability of Helicopter Rotor in Forward Flight: Results and Validation

Multiblade Transformation of SIMPACK Linear System Matrix

Transformed system Matrix”T” of system Matrix “A” using MATLAB program:

$\Omega = 1,\ B = 0.97\ ,\ \mu = 0.3\ ,\ \gamma = 12\ ,\ k_\beta = 0\ , \psi = 67.5$

```
T =

         0         0         0         0    1.0000         0         0         0
         0         0         0         0         0    1.0000         0         0
         0         0         0         0         0         0    1.0000         0
         0         0         0         0         0         0         0    1.0000
   -1.0000    0.0000    0.0000    0.0898   -1.3279    0.0000   -0.2738         0
   -0.5476    0.0635   -1.3914   -0.3872    0.0000   -1.3279   -2.0000    0.3872
   -0.0000    1.2644   -0.0635    0.3872   -0.5476    2.0000   -1.3279    0.3872
    0.0898   -0.3872    0.3872   -1.0000         0    0.1936    0.1936   -1.3279
```

Comparing the transformation result with analytical calculations:

$$T_{54} = \mu^2 \frac{\gamma}{8} B^2 \sin 2\psi = 0.0898$$

$$T_{62} = -\frac{k_\beta}{I\Omega^2} - \mu^2 \frac{\gamma}{16} B^2 \sin 4\psi = 0.0635$$

$$T_{63} = -\frac{\gamma}{8} B^4 - \frac{\gamma}{16} \mu^2 B^2 + \frac{\gamma}{16} \mu^2 B^2 \cos 4\psi = -1.3914$$

$$T_{64} = \mu \frac{\gamma}{6} B^3 \cos 2\psi = 0.3872$$

$$T_{72} = \frac{\gamma}{8} B^4 - \frac{\gamma}{16} \mu^2 B^2 + \frac{\gamma}{16} \mu^2 B^2 \cos 4\psi = 1.2644$$

$$T_{73} = -\frac{k_\beta}{I\Omega^2} + \mu^2 \frac{\gamma}{16} B^2 \sin 4\psi = -0.0635$$

$$T_{74} = \mu \frac{\gamma}{6} B^3 \sin 2\psi = 0.3872$$

$$T_{81} = \mu^2 \frac{\gamma}{8} B^2 \sin 2\psi = 0.0898$$

$$T_{82} = \mu \frac{\gamma}{6} B^3 \cos 2\psi = -0.3872$$

$$T_{83} = \mu \frac{\gamma}{6} B^3 \sin 2\psi = 0.3872$$

$$T_{84} = -1 - \frac{k_\beta}{I\Omega^2} = -1$$

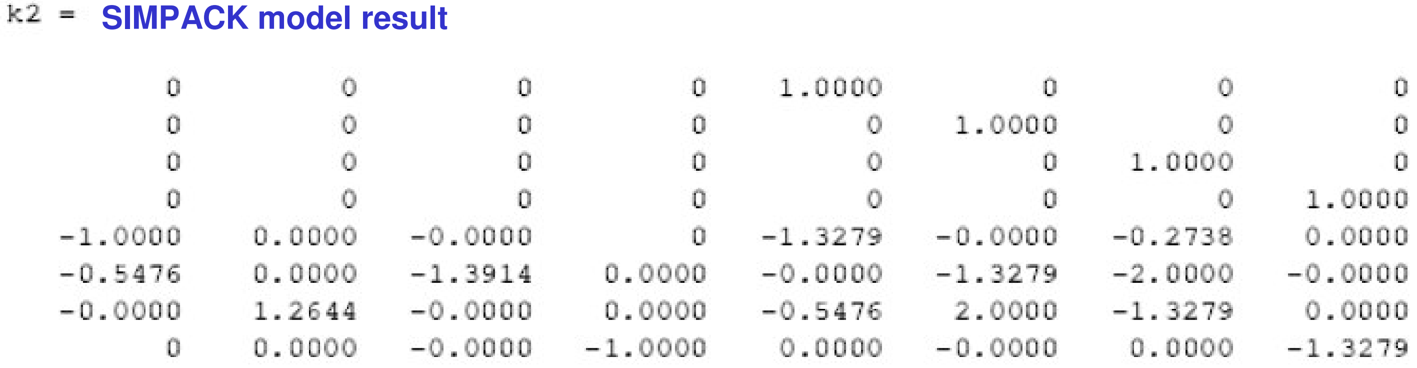
Flapping Stability of Helicopter Rotor in Forward Flight: Results and Validation

$$\frac{\sum_{0}^{2\pi}[T]_{\psi_i}}{n} \Rightarrow$$

time average: analytical result

$$\begin{bmatrix} \dot{\beta}_0 \\ \dot{\beta}_{1c} \\ \dot{\beta}_{1s} \\ \dot{\beta}_d \\ \ddot{\beta}_0 \\ \ddot{\beta}_{1c} \\ \ddot{\beta}_{1s} \\ \ddot{\beta}_d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1-\frac{k_\beta}{I\Omega^2} & 0 & 0 & 0 & -\frac{\gamma}{8}B^4 & 0 & -\mu\frac{\gamma}{12}B^3 & 0 \\ -\mu\frac{\gamma}{6}B^3 & -\frac{k_\beta}{I\Omega^2} & -\frac{\gamma}{8}B^4-\frac{\gamma}{16}\mu^2B^2 & 0 & 0 & -\frac{\gamma}{8}B^4 & -2 & 0 \\ 0 & \frac{\gamma}{8}B^4-\frac{\gamma}{16}\mu^2B^2 & -\frac{k_\beta}{I\Omega^2} & 0 & -\mu\frac{\gamma}{6}B^3 & 2 & -\frac{\gamma}{8}B^4 & 0 \\ 0 & 0 & 0 & -1-\frac{k_\beta}{I\Omega^2} & 0 & 0 & 0 & -\frac{\gamma}{8}B^4 \end{bmatrix} \cdot \begin{bmatrix} \beta_0 \\ \beta_{1c} \\ \beta_{1s} \\ \beta_d \\ \dot{\beta}_0 \\ \dot{\beta}_{1c} \\ \dot{\beta}_{1s} \\ \dot{\beta}_d \end{bmatrix}$$

$$\Omega = 1,\ B = 0.97\ ,\ \mu = 0.3\ ,\ \gamma = 12\ ,\ k_\beta = 0$$



SIMPACK model result

```
k2 =

         0         0         0         0    1.0000         0         0         0
         0         0         0         0         0    1.0000         0         0
         0         0         0         0         0         0    1.0000         0
         0         0         0         0         0         0         0    1.0000
   -1.0000    0.0000   -0.0000         0   -1.3279   -0.0000   -0.2738    0.0000
   -0.5476    0.0000   -1.3914    0.0000   -0.0000   -1.3279   -2.0000   -0.0000
   -0.0000    1.2644   -0.0000    0.0000   -0.5476    2.0000   -1.3279    0.0000
         0    0.0000   -0.0000   -1.0000    0.0000   -0.0000    0.0000   -1.3279
```

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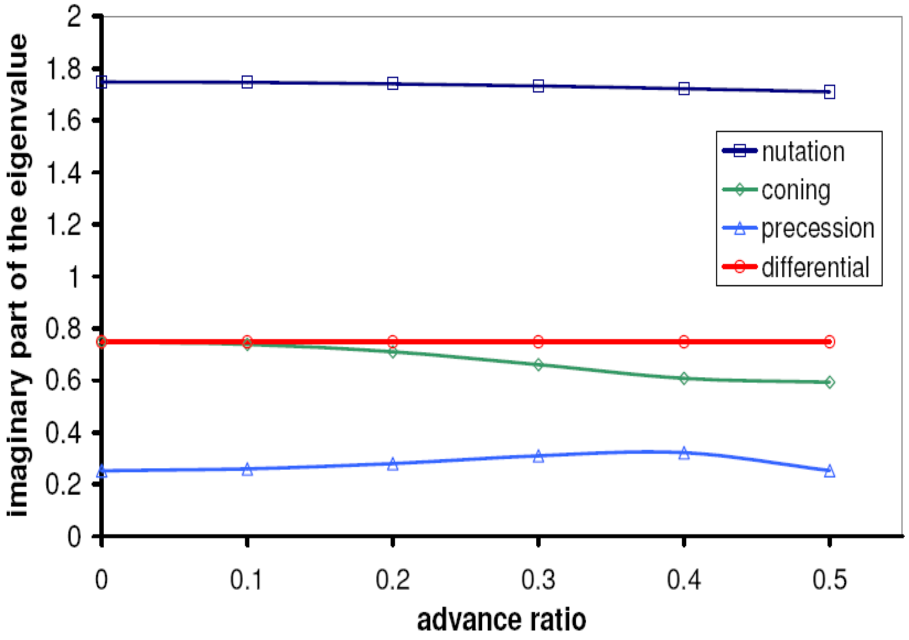
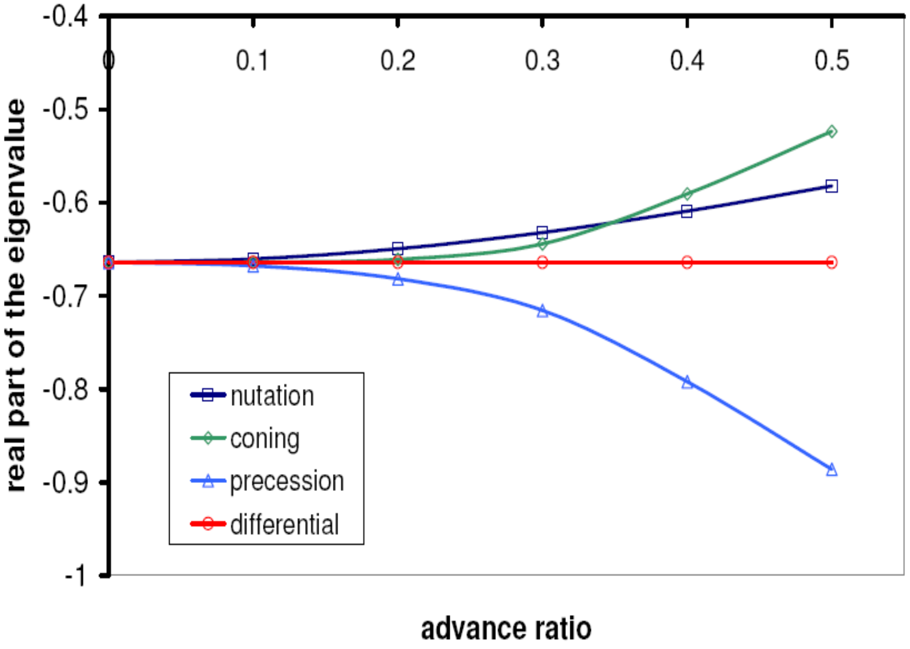
Flapping Stability of Helicopter Rotor in Forward Flight: Results and Validation

Flapping: eigenvalues of the transformed system matrix with constant coefficient approximation



Flapping: eigenvalues of the transformed system matrix with constant coefficient approximation



nutation/ progressive mode: higher frequency mode

precession/ regressive mode: lower frequency mode

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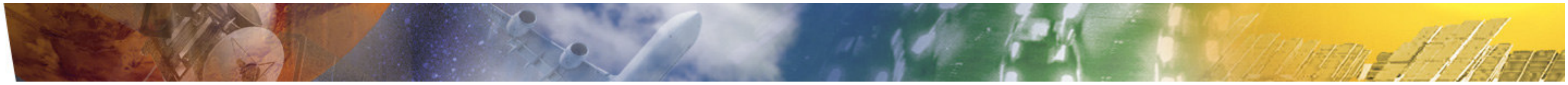

Conclusion

- Existing SIMPACK modeling options like „Substitution Variables“ and „Expression“ allows creating parametric model of a complex mathematical formulation, which can be used for example as force or time excitation inside SIMPACK model.
- SIMPACK interfaces with other codes like „MATLAB“ is a great option for multidisciplinary simulation to use the computational advantages of different codes.
- Simulation and stability analysis presented here is just a simple example to introduce the method, which can be used for a detailed helicopter model created inside SIMPACK.